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Abstract

We consider a simple variant of the standard real business cycle model in which shareholders hire a self-interested executive to manage the firm on their behalf. A generic family of compensation contracts similar to those employed in practice is studied. When compensation is convex in the firm's *own* dividend (or share price), a given increase in the firm's output generated by an additional unit of physical investment results in a more than proportional increase in the manager's income. Incentive contracts of sufficient yet modest convexity are shown to result in an indeterminate general equilibrium, one in which business cycles are driven by self-fulfilling fluctuations in the manager's expectations that are unrelated to the economy's fundamentals. Arbitrarily large fluctuations in macroeconomic variables may result. We also provide a theoretical justification for the proposed family of contracts by demonstrating that they yield first-best outcomes for specific parameter choices.

Key words: delegation, executive compensation, indeterminacy and instability

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1 Introduction

Executive compensation in public companies typically is provided in the form of three components, a cash salary (the wage and pension contributions), a bonus related to the firm's short term operating profit, and stock options (or other related forms of compensation based on the firm's share price). In seeking to create stronger links between pay and performance, the use of stock options, in particular, has emerged as the single largest ingredient of U.S. executive compensation. According to Hall and Murphy (2002), "in fiscal 1999, 94% of S&P 500 companies granted options to their top executives. Moreover, the grant-date value of stock options accounted for 47% of total pay for S&P 500 CEOs in 1999." CEOs of the largest U.S. companies frequently receive annual stock option awards that are on average larger than their salaries and bonuses combined.[1,2] Frydman and Jenter (2010) report that for the period 2000-2005, options and other long term incentive pay averaged 60% of total executive compensation; in 2008 the salary component had fallen to only 17% of average total pay.

Executive options contracts represent a particular instance of a highly non-linear convex style contract.[3] In this paper we demonstrate that convex executive pay practices, within the context of the separation of ownership and control in the modern corporation, may have dramatic, adverse business cycle consequences. In particular, we show that convex compensation contracts may give rise to generic sunspot equilibria in otherwise standard dynamic stochastic general equilibrium models. Sunspot equilibria (indeterminacy) formalize the notion that expectations not grounded

[1] See also Jensen and Murphy (1990); also, Shleifer and Vishny (1997) and Murphy (1999).

[2] In the past few years, restricted stock grants have begun to supplant strict call options as the largest category of executive compensation. Restricted stock grants are awards of stock to managers that cannot be sold until the end of a prespecified vesting period and/or until certain performance goals are met. The purchase price of the stock is preset at the grant date, and the executive typically receives the difference between the purchase price and the sale price of the stock at the conclusion of the vesting period. At the level of abstraction of this paper, such grants are nothing more than long term options with a time to maturity equal to the required vesting period. In actual practice, however, the executive incurs no tax liability in the vesting period (until he cashes out) whereas the value of options grants are immediately taxable as ordinary income at the initial grant date. Such favored tax treatment encourages the use of stock grants over pure options.

[3] To clarify the sense of an options contract being convex, consider a single call option with a payoff, c_T, at expiration of $c_T = \max\,\{0, q_T^e - E\}$ where T is the expiration date, q_T^e is the price of the underlying stock at expiration, and E is the exercise price. The payoff is piecewise linear and convex in the sense that if $q_T^e \leq E$, $c_T = 0$, and if $q_T^e > E$, $c_T = (q_T^e - E)$, the latter being representable as a line with unit slope over its region of definition.

A portfolio of N call options would have a diagonal payoff line that is much steeper (the slope would, in fact, be "N"). In this sense the payoff to N options is "more convex" than the payoff to one option: increases in q_T^e above E have a much greater monetary benefit to the owner of the calls. When a CEO is given a grant of 1,000,000 options the diagonal line becomes nearly vertical and convexity in the above sense becomes enormous.

in fundamentals may lead to behavior by which they are fulfilled. These equilibria may involve arbitrarily large fluctuations in macroeconomic variables even though production is characterized by constant returns to scale at the social as well as private level. As such, convex managerial compensation contracts provide an entirely new mechanism by which indeterminacy may arise in real (non-monetary) economies. An even more disturbing observation is that convex contracts may lead, under certain parameter configurations, to non-stationary behavior. Practically speaking this means that convex contracts may induce the self-interested manager to adopt investment policies that drive his firm's equilibrium capital stock to zero.[4]

Our focus on convex contracts derives from the fact that these contracts will also be shown to arise endogenously in our model context: the optimal contract, optimal in the sense of generating the first best allocation, is convex in the firms' aggregate free cash flow (dividends). This feature is necessary to align the incentives of managers and shareholders who are assumed to have different elasticities of intertermporal substitution and thus different preferences for intertemporal consumption allocations.[5] The optimal contract in this model does not, per se, generate equilibrium indeterminacy. We show, however, that the equilibrium indeterminacy discussed above can easily arise in the face of small deviations from the optimal contract. By small deviations we mean that the contract involves a slightly higher degree of convexity than is required to perfectly align incentives, or involves a fixed salary component that is added to the manager's incentive compensation, features that characterize actual compensation contracts. Nevertheless, while we derive the optimal contract in the model ultimately to justify our interest in convex compensation contracts in general, the focus of the analysis remains on the equilibrium consequences of a general class of convex compensation contracts which resemble contracts actually observed in practice.

To keep the analysis as simple as possible and to isolate the key source of equilibrium indeterminacy in the model, we assume that both consumer-worker-shareholders and managers have the

[4] Financial firms seem especially prone to lavishly convex compensation practices. We are reminded of the financial crises surrounding the collapse of LTCM. In the year preceding its bankruptcy, the partners took the deliberate decision to reduce the firm's capital, as a device for maximizing returns. More recently (2008) highly convex managerial compensation at various investment banks was observed in parallel with their bankruptcy. We view these compensation contracts as highly convex to either the firm's stock price or its free cash flow (or distributions to investors in the case of hedge funds).

[5] To say it differently, such a feature helps to align the stochastic discount factors of the managers and consumer-worker-shareholders.

same information. This assumption stands in contrast with standard principal-agent theory that focuses either on the effects of hidden information as to the manager's type or on the manager's hidden actions (effort). Our results apply more broadly, however, to contexts where consumer-shareholder-workers and managers have differing information regarding the economy's state variables.[6] We eschew this added generality for two reasons: First, it allows us to focus exclusively on the indeterminacy generating mechanism which is the same in either context. Second, for the model presented here, the presence or absence of indeterminacy is related only to the manager's elasticity of intertemporal substitution, and the terms of the contract given to him. Nothing else is relevant. Further augmenting the model to allow for hidden information regarding the manager's "type" or actions, while likely providing additional justifications for offering a convex contract, would not, per se, affect the presence or absence of indeterminacy.[7]

The earlier literature on equilibrium indeterminacy in dynamic models relies on a variety of other mechanisms; e.g., aggregate increasing returns, a difference between social and private returns to scale, a variable degree of competition, or monetary phenomena to generate multiple equilibria.[8] In our economy with delegated management and a convex executive compensation contract, the wedge between the actual return on capital and the return on capital as experienced by the manager is at the heart of the indeterminacy result. The power (degree of convexity) of the performance portion of the executive compensation contract tends to magnify the effective rate of return on capital from the manager's perspective. As a result, the expectation of a high return on capital may increase the income of the manager next period to such an extent that consumption smoothing considerations dictate a diminished level of investment today, thereby fulfilling the high return expectation. Nevertheless, our analytical and numerical results reveal that the degree of contract convexity required for indeterminacy is very low, especially so relative to a standard call options

[6] This particular information asymmetry is accommodated in Danthine and Donaldson (2010). From a macroeconomic perspective it is most relevant asymmetry.

[7] There is no unobservable effort decision on the manager's part in our model and there is no issue as to his concealed "type." The "Revelation Principle" thus does not pertain to our model. Rather, the information asymmetry, should we have elected to include it, concerns the inability of the shareholder-workers to infer the economy's state variables, in particular, its productivity shock. These are known only to the manager. If the manager is given the optimal contract, a first-best allocation will also be achieved under the information asymmetry. This contract is identical to the one presented in the full information equilibrium we emphasize.

[8] See, for example, the works of Benhabib and Farmer (1996), Wen (1998), Perli (1998), Benhabib, Meng and Nishimura (2000), Clarida, Galí and Gertler (2000), Woodford (2003), Jaimovich (2007) and Bilbiie (2008).

style incentive contract.

An outline of the paper is as follows: Section 2 describes the model and presents the family of compensation contracts we propose to study. It also details the precise circumstances under which equilibrium indeterminacy and instability arise. Section 3 provides an overview of the methodology by which an indeterminate equilibrium may be computed numerically and applies it to the study of the economy's business cycle characteristics. Section 4 provides a theoretical justification for the analysis of the proposed family of contracts by demonstrating that they yield first best outcomes for specific parameter choices. Section 5 concludes.

2 Convex Contracting in a General Equilibrium Model of Delegated Management

We focus on the context of a self-interested manager and the consumer-shareholder-workers on whose behalf he undertakes the firm's investment and hiring decisions in light of his compensation contract. We assume that there exists a continuum of measure 1 of identical consumer-worker-shareholders who consume a single good and supply homogenous labor services to a continuum of measure 1 of identical firms.[9] The consumer-shareholder-workers delegate the firm's management to a measure $\mu \in [0, 1]$ of managers who receive sufficient compensation to be willing to oversee the firm. We assume that once the manager agrees to operate the firm, he commits to it for the indefinite future. We further assume full information in the sense that both the consumer-worker-shareholder (principal) and the manager (agent) know the realization of all present and past variables.[10] Since there is no distortion in this economy, the first best can potentially be achieved. In the benchmark model, we suppose that the managers have no access to financial markets. With no opportunity to borrow or lend, they consume all of their income in each period. Guided by their compensation contract, they seek to smooth their consumption over time by undertaking the firm's investment

[9] We suppose that an equal number of workers participate in each firm.

[10] Delegating to the managers the firm's investment and hiring decisions makes sense even in an environment of full information if, say, there is a cost to shareholders of gathering and voting on the preferred investment and hiring plans (there would be unanimity). This cost could also be associated with information acquisition and borne either by a small measure of delegated managers or all the shareholder-workers. Under the optimal contract, whether the shareholders make the decision themselves or the managers behave in a self-directed way in light of their contract, the equilibrium allocation is the same.

and hiring decisions. In Appendix F, we extend the model to allow managers to buy or sell riskless bonds. This is done to show that the results to be presented here are not sensitive to the assumption that managers are excluded from financial markets.

We start by describing the environment surrounding the consumer-shareholder-worker, the firm and the manager and introduce the family of compensation contracts we propose to consider.[11] We then characterize the economy's equilibrium allocation and characterize the nature of equilibrium indeterminacy and instability. In Section 4, we then demonstrate that the same equilibrium is Pareto optimal when the contract parameters assume specific values.

2.1 The Model

2.1.1 The Representative Consumer-Worker-Shareholder

The representative consumer-worker-shareholder chooses processes for per-capita consumption c_t^s, the fraction of the time endowment, n_t^s, he wishes to work, his next period's investment in one period risk-free discount bonds, b_{t+1}^s, and his equity holdings $z_{t+1}^e(f)$ in firm f, to maximize his expected life-time utility

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(c_t^s)^{1-\eta_s}}{1-\eta_s}-B\frac{(n_t^s)^{1+\zeta}}{1+\zeta}\right)\right] \tag{1}$$

subject to his budget constraint

$$c_t^s+\int_0^1 q_t^e(f)\,z_{t+1}^e(f)\,df+q_t^b b_{t+1}^s=\int_0^1\left(q_t^e(f)+d_t(f)\right)z_t^e(f)\,df+w_t n_t^s+b_t^s, \tag{2}$$

in all periods. The subjective discount factor β satisfies $0<\beta<1$, the parameter η_s denotes the representative shareholder's relative risk aversion coefficient $(0<\eta_s<\infty)$ and ζ is the inverse of the Frisch elasticity of labor supply $(0\leq\zeta<\infty)$. Note that in the case of $\zeta=0$, the utility function in (1) reduces to the indivisible labor utility specification of Hansen (1985), while we obtain the case of fixed labor supply when $\zeta\to\infty$. Each consumer-worker-shareholder is endowed with one

[11] The model generalizes and refines the one found in Danthine and Donaldson (2008 a,b). The latter papers consider only optimal contracts and only the form they assume at equilibrium. Non-optimal contracts are not considered. In addition, the focus of these papers is to provide an explanation for the 'relative performance evaluation' and 'one-sided relative performance evaluation' puzzles discussed in the executive compensation literature. The present paper does not address these phenomena but focuses on the implications of a general class of contracts, and derives in Section 4 an optimal contract under more general conditions.

unit of time; the parameter $B > 0$ determines, in part, the fraction of that time endowment devoted to work. In (2), w_t denotes the competitive wage rate, q_t^b is the bond price, $q_t^e(f)$ is the share price of firm f, and $d_t(f)$ its associated dividend. We assume that each household holds a fully diversified portfolio of shares of each firm in equal proportions. We furthermore assume non-negativity constraints on all of the above variables.

The consumer-worker-shareholder chooses $\{c_t^s, n_t^s, z_{t+1}^e(f), b_{t+1}^s\}$ to maximize his welfare (1) subject to the budget constraint (2). This yields the standard first-order necessary conditions:

$$(c_t^s)^{-\eta_s} w_t = B(n_t^s)^{\zeta} \tag{3}$$

$$q_t^e(f) = E_t\left[\beta \frac{\left(c_{t+1}^s\right)^{-\eta_s}}{\left(c_t^s\right)^{-\eta_s}} \left(q_{t+1}^e(f) + d_{t+1}(f)\right)\right], \text{ and} \tag{4}$$

$$q_t^b = E_t\left[\beta \frac{\left(c_{t+1}^s\right)^{-\eta_s}}{\left(c_t^s\right)^{-\eta_s}}\right]. \tag{5}$$

2.1.2 Firms

On the production side, there is a continuum of measure one of identical, competitive firms. Firm $f \in [0,1]$ produces output via a standard constant returns to scale Cobb-Douglas production function:

$$y_t(f) = (k_t(f))^{\alpha} (n_t(f))^{1-\alpha} e^{\lambda_t} \tag{6}$$

with two inputs – capital, $k_t(f)$, and labor, $n_t(f)$ – and the current level of technology λ_t; the latter is assumed to be common to all firms and to follow a stationary process which we denote by $\lambda_{t+1} \sim dG(\lambda_{t+1}; \lambda_t)$. The evolution of the firm's capital stock, $k_t(f)$, follows:

$$k_{t+1}(f) = (1-\Omega) k_t(f) + i_t(f), \qquad k_0(f) = k_0 \text{ given,} \tag{7}$$

where $i_t(f)$ is the period t investment of firm f and Ω, $0 < \Omega < 1$ is the depreciation rate. The firm's dividend, $d_t(f)$, is in turn given by the free cash flows of the firm

$$d_t(f) = y_t(f) - w_t n_t(f) - i_t(f) - \mu g_t^m(f) \quad (8)$$

which amounts to the firm's income minus its wage bill, its investment expenditures and the managers' compensation $\mu g_t^m(f)$. Here, $g_t^m(f)$ denotes the per-capita compensation of the managers of firm f, and $\mu \in (0,1]$ denotes the measure of such managers working in firm f. In what follows we view managers as acting collegially, and thus use the words "manager" and "managers" interchangeably.

2.1.3 The Manager

At date 0, the representative manager of firm f decides whether or not to manage the firm. If he elects not to manage the firm, he receives a constant stream of reservation utility $\bar{u}^m$ in each period. If he does decide to manage the firm, he chooses processes for his consumption $c_t^m(f)$, hiring decisions $n_t(f)$, investment, $i_t(f)$, in the physical capital stock $k_t(f)$, and dividends $d_t(f)$ to maximize his expected utility

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \frac{(c_t^m(f))^{1-\eta_m}}{1-\eta_m}\right] \quad (9)$$

subject to the production function (6), the capital accumulation equation (7), the dividend equation (8), and the constraints

$$c_t^m(f) \leq g_t^m(f) \quad (10)$$
$$c_t^m(f), n_t(f), k_t(f), y_t(f) \geq 0.$$

The manager thus chooses to operate the firm provided that his participation constraint

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \frac{(c_t^m(f))^{1-\eta_m}}{1-\eta_m}\right] \geq \frac{\bar{u}^m}{1-\beta} \quad (11)$$

is also satisfied.

Managers need not be risk neutral nor have the same degree of relative risk aversion, η_m, as the consumer-worker-shareholders. For simplicity, we assume managers do not confront a labor-leisure trade-off.

The manager's budget constraint (10) states that his consumption can be no larger than his compensation, $g_t^m(f)$. As noted earlier, we assume that the manager does not participate in the capital markets. It is realistic to presume the manager is banned from trading the equity issued by the firm he manages. Not only does this rule protect shareholders from insider trading, but it also prevents the manager from using financial markets to mitigate the force of his contract. Restrictions on the ability of the executives to assume short positions in the stock of their own firms, or to adjust their long positions, are commonplace. It is more controversial however to assume that the manager cannot take a position in the risk free asset, although this assumption is common in the partial equilibrium contracting literature. We maintain this assumption here, though we relax it in Appendix F, where we demonstrate that our main results are, in fact, reinforced when managers are allowed to trade a risk-free bond.[12]

We consider the general family of contracts

$$g^m(w_t n_t, d_t, d_t(f)) = A + \varphi\left[(\delta w_t n_t + d_t)^\gamma + d_t(f) - d_t\right]^\theta \tag{12}$$

with constant coefficients $A \geq 0$, $\varphi \geq 0$, $0 \leq \delta \leq 1$, and constant exponents $\gamma > 0$ and $\theta > 0$. The expression $w_t n_t$ denotes the equilibrium aggregate wage bill; and d_t the equilibrium aggregate dividend; δ represents the relative compensation weight applied to the wage bill vis-a-vis the dividend and φ the overall compensation scale parameter. Parameters γ and θ determine the degree of the convexity of the compensation contract.[13]

Contracts of the form (12) have the flavor of the three component contracts mentioned in the

[12] Appendix F is available online from the authors' websites.

[13] Contracts of the form (12) are also optimal within our model context, provided appropriate restrictions on the values of the parameters $A, \delta, \varphi, \gamma, \theta$ are imposed. This is the subject of Section 4.

introduction. For instance, if $\theta = 2$, contract (12) can be rewritten as

$$g^m(w_t n_t, d_t, d_t(f)) = A + \varphi\left[(\delta w_t n_t + d_t)^{2\gamma} + 2(\delta w_t n_t + d_t)^{\gamma}(d_t(f) - d_t) + (d_t(f) - d_t)^2\right]$$

where $A + \varphi(\delta w_t n_t + d_t)^{2\gamma}$ denotes a constant wage payment, $2\varphi(\delta w_t n_t + d_t)^{\gamma}(d_t(f) - d_t)$ denotes a variable 'bonus' component, that is proportional to the firm's current dividend (or free cash flow), and $\varphi(d_t(f) - d_t)^2$ approximates an 'option' component.[14, 15]

Note that when focusing on first-order approximations, as we do in the next section, the equity price q_t^e can be substituted for the dividend without loss of optimality (although the coefficient φ may need to be modified accordingly). To a first-order approximation, we may then express the incentive component in terms of the manager's own firm stock price instead of the dividend.

How large is the degree of convexity in the typical compensation contract? Gabaix and Landier (2008) carefully study the link between firms' total market value (debt plus equity) and total compensation for the 1,000 highest paid CEOs in the U.S., over the period from 1992 to 2004. Their compensation measure includes the following components: salary, bonus, restricted stock grants and Black-Scholes values of stock options granted. Using panel regressions, they find that the elasticity of CEO compensation to the firms' total market value is slightly above 1 (see their Table 2). While they do not formally reject an elasticity of 1 at the 5% confidence level, the point estimates lie above 1 in all specifications and are in some cases significantly larger than 1, at the 10% confidence level. Using the more aggregated compensation index of Jensen, Murphy, and Wruck (2004), which is based on all CEOs included in the S&P 500, they estimate that an increase of 1% in the mean of the largest 500 firms' asset market values increases CEO compensation by 1.14% on average in the 1970-2003 sample (see their Table 3).[16] Their Figure 1 suggests that this elasticity is significantly larger in the 1990-2000 period. While we will focus our analysis on moderate levels of contract convexity, it is important to note that this convexity can easily be very large when the

[14] The third term appears more akin to an option's payoff when we recognize that within this model class, $d_t(f)$ is roughly proportional to $q_t^e(f)$ and d_t is roughly proportional to q_t^e, the value of the market portfolio. Accordingly, the manager's long term incentive pay is proportional to how well his own firm's stock performs relative to the market if $d_t(f) > d_t$.

[15] By a constant wage payment we mean a salary component that is independent of the firm's own operating income.

[16] We will use this latter estimate to determine a reasonable value of θ in our simulation work to follow.

compensation involves many call options.[17]

The measure μ of managers who work in firm f choose $\{k_{t+1}(f), n_t(f), c_t^m(f), d_t(f)\}$ to maximize their utility (9) subject to the restrictions (6)–(8), (10), (11) and the compensation contract (12). We will assume that φ is sufficiently large for each manager's participation constraint (11) to be satisfied.[18] The Lagrangian for the problem of the representative manager in firm f can be expressed as

$$\begin{aligned}
\mathcal{L} \;=\; & E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{(c_t^m(f))^{1-\eta_m}}{1-\eta_m} \right. \right. \\
& + \Lambda_{1t}(f) \Big[(k_t(f))^{\alpha} (n_t(f))^{1-\alpha} e^{\lambda_t} - w_t n_t(f) - k_{t+1}(f) + (1-\Omega) k_t(f) \\
& - \mu \left(A + \varphi \left[(\delta w_t n_t + d_t)^{\gamma} + d_t(f) - d_t \right]^{\theta} \right) - d_t(f) \Big] \\
& \left. \left. + \Lambda_{2t}(f) \Big[A + \varphi \left[(\delta w_t n_t + d_t)^{\gamma} + d_t(f) - d_t \right]^{\theta} - c_t^m(f) \Big] \right) \right\}.
\end{aligned}$$

The necessary first-order conditions with respect to $c_t^m(f), n_t(f), k_{t+1}(f), d_t(f)$ imply

$$\Lambda_{2t}(f) = (c_t^m(f))^{-\eta_m} \tag{13}$$

$$w_t = (1-\alpha) y_t(f) / n_t(f) \tag{14}$$

$$\Lambda_{1t}(f) = E_t \{ \beta \Lambda_{1t+1}(f) [1 - \Omega + \alpha y_{t+1}(f) / k_{t+1}(f)] \} \tag{15}$$

$$\Lambda_{1t}(f) = \Lambda_{2t}(f) \frac{x_t(f)}{1 + \mu x_t(f)} \tag{16}$$

for all f, where

$$x_t(f) \equiv \frac{\partial g^m (w_t n_t, d_t, d_t(f))}{\partial d_t(f)} = \theta \varphi \left[(\delta w_t n_t + d_t)^{\gamma} + d_t(f) - d_t \right]^{\theta - 1}. \tag{17}$$

The variable $x_t(f)$ will turn out to be critical. It denotes the marginal contribution to the manager's compensation of increasing the firm's dividend by one unit.

[17] To put our claim in perspective, consider a standard call options clarify contract where the degree of convexity is measured, using the Black-Scholes call valuation formulae, by gamma (Γ). To award a manager a call option on his firm's stock is directly analogous to granting him a compensation contract of the form (12) with $\theta > 1$. Typically, the strike price of an options award is set equal to the then-prevailing stock price. The gamma of a long position in a call option is always positive and reaches a maximum under this circumstance (for given volatility, time to expiration, etc.).

[18] See the discussion in Appendix C available online from the authors' websites.

Using (13) and (16) to solve for the Lagrange multipliers $\Lambda_{1t}(f)$ and $\Lambda_{2t}(f)$, we can rewrite (15) as:

$$(c_t^m(f))^{-\eta_m} \frac{x_t(f)}{1+\mu x_t(f)} = E_t\left[\beta\left(c_{t+1}^m(f)\right)^{-\eta_m} \frac{x_{t+1}(f)}{1+\mu x_{t+1}(f)} r_{t+1}\right] \tag{18}$$

where r_t, the real return to physical capital, is defined by

$$r_t \equiv (1-\Omega) + \alpha y_t / k_t, \qquad \text{and}$$

$$x_t(f) = \theta \varphi^{1/\theta} \left(c_t^m(f) - A\right)^{\frac{\theta-1}{\theta}}. \tag{19}$$

Equality (19) is derived using (17), contract (12) and the fact that $c_t^m(f) = g_t^m(f)$.

2.1.4 Equilibrium

Since all firms are identical, all managers face the same constraints, solve the same problem, and thus make the same decisions. It follows that $c_t^m(f) = c_t^m \equiv \int_0^1 c_t^m(f)\, df$, $y_t(f) = y_t \equiv \int_0^1 y_t(f)\, df$ and so on for all f. As a consequence, the same constraints and first-order conditions hold without the index f. Equilibrium is then a set of processes $\{c_t^s, c_t^m, n_t^s, n_t^f, b_t^s, z_t^e, i_t, y_t, k_t, r_t, w_t, q_t^b, q_t^e, d_t, x_t\}$ such that:

1. The first-order conditions (3)–(5), (14), and (18) are satisfied together with constraints (2), (6)–(8), (10), and (11) all holding with equality, and the transversality condition is satisfied, $\lim_{t\to\infty} \beta^t (c_t^m)^{-\eta_m} k_{t+1} = 0$, for any given initial k_0.

2. The labor, goods and capital markets clear: $n_t^s = n_t$; $y_t = c_t^s + \mu c_t^m + i_t$; investors hold all outstanding equity shares, $z_t^e = 1$, and all other assets (one period bonds) are in zero net supply, $b_t^s = 0$.

These equilibrium conditions can be summarized as follows. Consumption of the manager and of the consumer-worker-shareholders depends on labor income and dividends[19]

$$c_t^m = A + \varphi (\delta w_t n_t + d_t)^{\gamma\theta} \quad (20)$$

$$c_t^s = w_t n_t + d_t, \quad (21)$$

where dividends, in turn, relate to income and investment according to

$$d_t = y_t - w_t n_t - i_t - \mu \left(A + \varphi (\delta w_t n_t + d_t)^{\gamma\theta} \right). \quad (22)$$

The production function yields

$$y_t = k_t^{\alpha} n_t^{1-\alpha} e^{\lambda_t}, \quad (23)$$

so that the real wage and the return on capital, r_t, are given, respectively, by

$$w_t = (1-\alpha)(y_t/n_t), \text{ and} \quad (24)$$

$$r_t \equiv \alpha (y_t/k_t) + 1 - \Omega. \quad (25)$$

The intratemporal first-order condition for the shareholder-worker's optimal consumption-leisure decision is

$$(c_t^s)^{-\eta_s} w_t = B n_t^{\zeta}. \quad (26)$$

While the above equations are all a-temporal, the equations determining the model's intertemporal dynamics are the capital accumulation equation

$$k_{t+1} = (1-\Omega) k_t + i_t \quad (27)$$

and the Euler equation for the optimal intertemporal allocation of the manager's consumption

$$(c_t^m)^{-\eta_m} \frac{x_t}{1+\mu x_t} = E_t \left[\beta \left(c_{t+1}^m \right)^{-\eta_m} \frac{x_{t+1}}{1+\mu x_{t+1}} r_{t+1} \right] \quad (28)$$

[19] Expression (20) is obtained from (10) and contract (12), recognizing that in equilibrium $d_t(f) = d_t$, while expression (21) follows from (2), noting that $b_t^s = 0$ and $z_t^e = 1$ in equilibrium.

where x_t is given in (19).[20]

We continue to assume that φ is large enough for the manager's participation constraint (11) to be satisfied, a restriction that does not affect the results to follow in any way. Equations (19)–(28), and their log-linearized counterparts, given below, form the basis of the analysis to follow.[21]

2.2 Approximating the Decentralized Equilibrium around the Deterministic Steady State

As we now show, the degree of convexity of the manager's contract has first order effects on the equilibrium dynamics. In particular, the convexity of the contract is crucial to determining whether the general equilibrium is unique or whether it exists at all. A global analysis of the existence and uniqueness of the general equilibrium of this model is beyond the scope of this paper. Instead, we focus here on the local analysis of the equilibrium dynamics around the deterministic steady state in the face of small enough exogenous disturbances.[22] Assuming the exogenous variable λ_t is bounded, and denoting the steady-state value of a variable with an overhead bar and the log-deviations from that steady-state value with a $\hat{}$, we can characterize the model's approximate dynamics by the following log-linearized equilibrium conditions:

$$\hat{c}_t^m = \Xi\left[\delta\omega\hat{y}_t + (1-\omega)\hat{d}_t\right], \qquad \text{where } \omega \equiv \frac{\bar{w}\bar{n}}{\bar{w}\bar{n}+\bar{d}},\ \Xi \equiv \frac{\gamma\theta\left(1-A/\bar{c}^m\right)}{\delta\omega+1-\omega} > 0 \tag{29}$$

$$\hat{c}_t^s = \omega\hat{y}_t + (1-\omega)\hat{d}_t \tag{30}$$

$$\Omega\frac{\bar{k}}{\bar{y}}\hat{\imath}_t = \left(\alpha - \mu\frac{\bar{c}^m}{\bar{y}}\Xi\delta\omega\right)\hat{y}_t - \left(\frac{\bar{d}}{\bar{y}} + \mu\frac{\bar{c}^m}{\bar{y}}\Xi(1-\omega)\right)\hat{d}_t \tag{31}$$

$$\hat{y}_t = \hat{\lambda}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t \tag{32}$$

$$\hat{w}_t = \hat{y}_t - \hat{n}_t \tag{33}$$

$$\hat{r}_t = (1-\beta(1-\Omega))\left(\hat{y}_t - \hat{k}_t\right) \tag{34}$$

$$\hat{w}_t = \eta_s\hat{c}_t^s + \zeta\hat{n}_t \tag{35}$$

[20] This equation determines the optimal intertemporal allocation of the manager's consumption even though managers do not have access to financial markets.

[21] Note that equations (4) and (5) determine asset prices q_t^e and q_t^b. We omit these equations here as asset prices are residual (non-state) variables for our class of models.

[22] The steady state in this economy is defined as the solution to the following set of equations: $\bar{c}^m = A + \varphi\left(\delta\bar{w}\bar{n}+\bar{d}\right)^{\gamma\theta}$; $\bar{w}\bar{n} = (1-\alpha)\bar{y}$; $\bar{y} = \bar{k}^\alpha\bar{n}^{1-\alpha}$; $\Omega\bar{k} = \bar{\imath}$; $\bar{x} = \theta\varphi^{1/\theta}\left(\bar{c}^m - A\right)^{(\theta-1)/\theta}$; $\bar{r} = \alpha\bar{y}/\bar{k} + 1 - \Omega$; $\beta^{-1} = \bar{r}$; $\bar{c}^s = \bar{w}\bar{n} + \bar{d}$ and $\left(\bar{c}^s\right)^{-\eta_s}\bar{w} = B\bar{n}^\zeta$ and $\bar{y} = \bar{c}^s + \bar{\imath} + \mu\bar{c}^m$. It follows that $\bar{k} = \bar{y}/\left(\beta^{-1} - 1 + \Omega\right)$.

$$\hat{k}_{t+1} = (1-\Omega)\,\hat{k}_t + \Omega \hat{\imath}_t, \tag{36}$$

and the log-linearized Euler equation for the manager's consumption[23]

$$\hat{c}_t^m = E_t \hat{c}_{t+1}^m - \psi^{-1} E_t \hat{r}_{t+1} \tag{37}$$

where

$$\psi \equiv \eta_m - \frac{\theta - 1}{\theta} \frac{1}{(1 - A/\bar{c}^m)(1 + \mu \bar{x})} \tag{38}$$

corresponds to the manager's coefficient of relative risk aversion adjusted for features of the incentive contract, such as its degree of convexity θ, and the fraction $A/\bar{c}^m$ of the manager's compensation that is fixed ($0 \leq A/\bar{c}^m < 1$).[24] As we will see below, ψ will turn out to be a key coefficient for the model's dynamics.

2.3 Indeterminacy: The Intuition

To understand how self-fulfilling fluctuations may arise in this economy, it is important to note that in equation (37), the coefficient ψ^{-1} represents the elasticity of intertemporal substitution for the manager's consumption in response to changes in the manager's *personal* rate of return on investment, i.e., the effective rate of return from the manager's point of view. That rate of return represents not only the additional output generated by another unit of investment in physical capital, but also the additional compensation distributed to the manager as a result of that additional unit of output. As indicated in (38), the degree of convexity of the incentive contract, θ, is a key determinant of the manager's effective intertemporal elasticity of substitution, and that for θ sufficiently larger than 1, ψ^{-1} may even be negative. As argued below, a negative ψ can imply an indeterminate equilibrium, so that economic fluctuations may result from self-fulfilling manager's expectations.

For comparison purposes, let us first explore the case where $\theta = 1$, a linear contract, so that $\psi = \eta_m$. For these circumstances, equation (37) reduces to the same log-linearized consumption

[23] Equation (37) is obtained after linearizing (19), and substituting for $\hat{x}_t = \left(\frac{\theta-1}{\theta}\right) \frac{\bar{c}^m}{\bar{c}^m - A} \hat{c}_t^m$.

[24] Equations (29) through (37) omit approximation error terms of second order or smaller.

Euler equation as would be obtained for the standard representative agent problem: $\hat{c}_t^m = E_t \hat{c}_{t+1}^m - \eta_m^{-1} E_t \hat{r}_{t+1}$. Here date t consumption responds negatively to increases in the expected rate of return (for given expected future consumption), and the response coefficient is the elasticity of intertemporal substitution. Assume that the manager suddenly expects a higher rate of return on capital next period, $\hat{r}_{t+1}$, than would be justified by fundamentals. In this case $\left(E_t \hat{c}_{t+1}^m - \hat{c}_t^m\right)$ must increase or $\hat{c}_t^m$ must get smaller, which can occur only if the agent saves more and so simultaneously $\hat{c}_{t+1}^m$ must increase. This can only happen by having the manager invest in future capital stock so that $\hat{k}_{t+1}$ also increases. The increase in the capital stock causes the marginal product of capital to drop (so that $\hat{r}_{t+1}$ declines). As a result, expectations of a higher return on capital cannot be fulfilled and there is no supportable equilibrium indeterminacy.

In the case of a convex compensation function $(\theta > 1)$, a given increase in the firm's output generated by an additional unit of physical investment results in a more than proportional increase in the manager's income. Let us consider the manager's contract with convexity sufficiently larger than 1 to guarantee that ψ is negative. In that case, suppose that the manager has the belief (unrelated to fundamentals) that his own personal return will be "high" next period. The perception of a high income next period will lead him – in the interest of consumption smoothing – to consume *more* today, and thus to reduce his investment today. The lower investment leads to a higher rate of return on capital which confirms the manager's belief of a high personal rate of return.

In general, the larger the convexity of the compensation contract, the more likely ψ^{-1} is to be negative, and hence the more likely the manager will increase his consumption and lower investment in the firm in response to his belief of an increase in the rate of return. It follows that the more convex the executive contract, the more likely the general equilibrium to be indeterminate, so that business cycles can be driven by self-fulfilling fluctuations in the manager's expectations. In contrast, with low convexity in the manager's contract $(\theta \leq 1)$, the increase in the return on capital is smaller from the manager's perspective. In this case, the contract works against the manager's "personal return expectations" being fulfilled.

An inspection of (38) also reveals that the lower the manager's risk aversion, η_m, the more likely ψ is to be negative and hence the more likely equilibrium indeterminacy will arise $(\partial\psi/\partial\eta_m = 1 > 0)$. In the extreme case of a risk-neutral manager $(\eta_m = 0)$, any convexity of the incentive contract

$(\theta > 1)$, no matter how small, implies a negative coefficient ψ, and hence can result in an indeterminate equilibrium. Similarly, for any convex contract $(\theta > 1)$, the larger the constant salary component of the executive contract, A, the more likely ψ will be negative, and, again, the more likely equilibrium indeterminacy can arise. By making the manager's compensation less volatile, the higher fixed salary component reduces the magnitude of the incentives part of the contract necessary to generate indeterminacy. This intuition is formalized below.

2.4 Indeterminacy and Instability in the Case of Fixed Labor Supply: An Analytical Characterization

We now determine the regions of the parameter space in which the model dynamics around the deterministic steady state yields (i) a unique bounded equilibrium, (ii) an indeterminate equilibrium so that an infinite number of bounded equilibria are consistent with the model's equations, or (iii) no bounded equilibrium so that the model's dynamics can only result in explosive paths. To derive analytical results, we consider a specification of (1) with a fixed labor supply $(\zeta \to \infty)$. It can be demonstrated numerically that none of the results presented here are affected by this assumption. In particular, the region of equilibrium indeterminacy is independent of the value of ζ.

After combining the linearized equilibrium conditions (29)–(38), the model's local dynamics can be summarized by the following two equations:

$$E_t \hat{c}^m_{t+1} = \hat{c}^m_t - \psi^{-1}(1-\alpha)(1-\beta(1-\Omega))\hat{k}_{t+1} + exog_t \tag{39}$$

$$\hat{k}_{t+1} = B_{21}\hat{c}^m_t + B_{22}\hat{k}_t + exog_t \tag{40}$$

where $exog_t$ denotes exogenous terms that depend on current and on expected future realizations of the productivity shock λ_t, and[25]

$$B_{21} = -\left(\frac{1}{(1-\omega)\,\Xi}\frac{\bar{d}}{\bar{k}} + \mu\frac{\bar{c}^m}{\bar{k}}\right) < 0 \tag{41}$$

$$B_{22} = \left(\beta^{-1} - (1-\Omega)\right)(1-\alpha)\,\delta + (1-\Omega)(1-\alpha) + \alpha\beta^{-1} > 0. \tag{42}$$

[25] Equation (39) is obtained by combining (37) and (34), using (32) to substitute for $\hat{y}_t$, and noting from (35) that $\hat{n}_t = 0$ in the case that $\zeta \to \infty$. To obtain (40), we first use (31) to express $\hat{\imath}_t$ as a function of $\hat{c}^m_t$, $\hat{k}_t$, and λ_t exploiting (29) to solve for $\hat{d}_t$, and (32) to eliminate $\hat{y}_t$, and then combine the resulting expression with (36).

Since B_{22} is increasing in δ, there exists a threshold

$$\delta^* \equiv 1 - \frac{\beta^{-1} - 1}{\left(\beta^{-1} - 1 + \Omega\right)(1 - \alpha)} < 1 \tag{43}$$

such that[26]

$$B_{22} < 1 \text{ if and only if } \delta < \delta^*.$$

The characterization of the regions of determinacy, indeterminacy and instability of the equilibrium resulting from the dynamic system (39)-(40) can be summarized as follows:

Theorem 1 *In the case of a fixed labor supply ($\zeta \to \infty$), the linearized model admits:*
(i) an indeterminate *equilibrium (i.e., a continuum of bounded solutions) if and only if the following conditions are jointly satisfied:*

$$\begin{aligned} \psi &< \psi^* \equiv \frac{(1-\alpha)(1-\beta(1-\Omega)) B_{21}}{2(B_{22}+1)} < 0 \\ \delta &< \delta^*, \end{aligned} \tag{44}$$

(ii) an unstable *equilibrium (i.e., no bounded solutions) if and only if (44) holds and $\delta > \delta^*$,*
(iii) a determinate *equilibrium (i.e., a unique bounded solution) if and only if $\psi \geq \psi^*$.*

Proof. See Appendix A. ■

A sufficient condition for a unique bounded equilibrium is $\psi > 0$. This theorem states that indeterminacy or instability (i.e., no bounded solution) arises provided that ψ is sufficiently negative. As we show below, this can easily occur with convex compensation contracts for the manager, and even for moderate degrees of convexity. In this case, the equilibrium is indeterminate for δ sufficiently small ($\delta < \delta^*$) and unstable for $\delta > \delta^*$.

To have some sense of the relevance of these conditions, we calibrate the quarterly subjective discount factor, β, at 0.99, the capital share of output, α, at 0.36 and the quarterly capital depreciation rate, Ω, at 0.025. These are values commonly used in the literature (see Section 3.1 for more details on the model calibration). The implied critical value $\delta^* = 0.55$. In addition, we assume that

[26] Note in particular that when $\delta = 1$, $B_{22} = \beta^{-1} > 1$.

consumer-worker-shareholders have log utility in consumption, so that $\eta_s = 1$, and set the measure of managers, μ, at 0.05. While the percentage of executive, administrative, and managerial occupations in the labor force amounts to 17%, we assume that less than a third of that makes actual hiring and investment decisions for the firm. (In any case, the particular value of μ is not critical for the results discussed below). If we furthermore assume that consumer-shareholder-workers and managers have the same consumption to income ratio in steady state, the above calibration implies that $\omega = 0.90$, which corresponds very closely to the average ratio obtained in U.S. national income data over the period 1947-2009.[27]

Figure 1 represents the regions of determinacy, indeterminacy and instability for various values of η_m and the parameters characterizing the incentive contract, i.e., the degree of overall convexity θ, the relative magnitude of the aggregate wage bill δ and the size of the fixed payment A as a fraction of the managers' steady-state consumption. We set γ, the degree of convexity in the aggregate wage bill and aggregate dividends equal to 1. Different values for this parameter would however not change any of our results in regards to the region of equilibrium determinacy. The boundary for the region of determinacy in the $(A/\bar{c}^m, \theta)$ space remains largely the same for the different values of δ represented in the two columns of Figure 1. With δ sufficiently low, the model exhibits local indeterminacy when the convexity of the contract θ rises. For $\delta = 1$, even a mildly convex managerial contract ($\theta > 1$) can lead to an explosive general equilibrium in the economy.

To get some intuition for this result, suppose that we start at the steady state and that there is no fundamental shock, i.e., $\lambda_t = 0$ for all t. Three possible scenarios may ensue:

Case 1: determinacy ($\psi > \psi^*$)

Suppose that the manager's compensation contract is linear or concave ($\theta \leq 1$) and that agents observe an unexpected sunspot ν_0 shock at date 0, which leads managers to consume more than in steady state ($\hat{c}_0^m > 0$). Since the initial capital stock is fixed at $\bar{k}$, the linearized equation for the capital stock, (40), implies that the capital stock in the next period will need to decrease ($\hat{k}_1 < 0$) because managers now consume more and invest less in physical capital. According to

[27] The ratio of employees' compensation over the sum of employees' compensation and corporate profits (after corporate tax, with IVA (investment valuation adjustment) and CCAdj (capital consumption adjustment) amounts to 0.91.

(39), high consumption by managers at date 0 and lower capital stock at date 1, leads to even higher managerial consumption in period 1 if ψ is large enough. This, in turn, causes a further drop in the capital stock and an increase in consumption by managers in period 2: $E_t\hat{c}_2^m > E_t\hat{c}_1^m > c_0$, and so on. In the case of $\psi > \psi^*$ the presence of a sunspot is inconsistent with a bounded equilibrium. Therefore, if a bounded equilibrium exists, it must be unique.

Case 2: indeterminacy ($\psi < \psi^* < 0$ and $\delta < \delta^*$)

Suppose instead that $\psi < \psi^* < 0$ (which occurs with a sufficiently convex compensation contract) and agents again observe an unexpected sunspot ν_0 at date 0, which leads managers to consume more at that time. Again, since the initial capital stock is fixed at its steady state value, equation (40) implies that the capital stock must decrease.

But with $\psi < \psi^*$ equations (39) and (40) imply that the increase in managerial consumption at date 0 combined with the lower capital stock at date 1, leads the manager to eventually consume less and accumulate more capital so that the economy reverts back to the steady state. This process leads to a stationary path for the manager's consumption and for the capital stock. If $\psi < \psi^*$, sunspot shocks are therefore consistent with a bounded equilibrium and there exists an infinite number of such bounded equilibria satisfying the model's restrictions, including some equilibria with arbitrary large fluctuations, as ν_t itself can be arbitrarily large.

Case 3: instability ($\psi < \psi^*$ and $\delta > \delta^*$)

Suppose again that $\psi < \psi^*$ and that δ exceeds its critical value so that $B_{22} > 1$. Consider a productivity or sunspot shock at date 0, which leads managers to invest less. Again, the capital stock at date 1 must fall below its steady state value. According to equation (39), if $\psi < \psi^*$ the manager's consumption in period 1 tends to be lower than in period 0, which according to equation (40), tends to bring the future capital closer to its steady state value. However, with $B_{22} > 1$, the date 1 deviation of the capital stock is amplified. It follows that the capital stock embarks on an explosive (or implosive) trajectory, eventually exiting any neighborhood of the steady state. This, in turn, results in an explosive evolution of the manager's consumption. Hence the model admits no bounded solution.

2.5 Indeterminacy and Instability for General Labor Supply

We use a numerical solution to show that the regions of determinacy remain the same when the labor supply is elastic although the split between indeterminacy and instability regions depends on the value of the Frisch elasticity of labor supply ζ^{-1} as well as the fraction δ of executive compensation related to aggregate labor income. Figures 2 and 3 show determinacy, indeterminacy, and instability regions in the $(A/\bar{c}^m, \theta)$ space, for different values of ζ.[28] In Figure 2, we set $\zeta^{-1} = 0.5$ to match the Frisch elasticity of labor supply often found in microeconomic studies, while in Figure 3, $\zeta^{-1} \to \infty$ consistently with the labor supply found in Hansen (1985). Once again, with sufficiently low δ, the convex executive compensation contract can easily generate an indeterminate equilibrium. Instead when δ is sufficiently large, the convex contract results in an explosive equilibrium dynamics.

3 Computing Equilibria

Here, we further explore the economy of Section 2 to see if it possesses reasonable business cycle characteristics.[29] Our numerical work is guided by three questions: (1) Are sunspot shocks fully harmonious with a productivity shock in the sense of the addition of the former not compromising the overall model's performance with regard to replicating the basic stylized facts of the business cycle? (2) In conjunction with a standard technology shock, does the addition of a sunspot shock enhance the explanatory power of the model along any specific dimensions? Lastly, (3) is it possible to generate a business cycle with the observed properties on the basis of sunspot shocks alone? If this is the case it becomes difficult to separate out the sources of business cycle fluctuations. For those skeptical of the notion of a productivity disturbance, affirmative answers to these questions would justify the model's reduced reliance on technology shocks as the sole economic driver. It has the less attractive implication, however, of suggesting that future macroeconomic volatility may not be forecastable since it may in part be determined by pure belief shocks. Our numerical strategy, as detailed in Appendix D, follows Sims (2000) and Lubik and Schorfheide (2003).[30]

[28] The interpretation of the experiments being undertaken in Figures 2 and 3 is exactly the same as for Figure 1.

[29] While proving nothing, a reasonable replication of the business cycle stylized facts nevertheless lends support for our overall modeling of sunspot equilibria.

[30] Appendix D is available online from the authors'websites.

3.1 Calibration

We divide the model's parameters into two groups. Values for the first group are obtained using standard calibrations, while the second set of values are estimated using GMM techniques. Whatever methodology is used, the resulting parameter values are chosen consistent with the model being simulated at quarterly frequencies. We first discuss the calibrated parameters.

The quarterly subjective discount factor, β is fixed at 0.99, yielding a steady state risk free rate of return of 4% per year. Capital's share of output, α, is chosen to be 0.36 (see Cooley and Prescott (1995) for a discussion) and the quarterly capital depreciation rate, Ω, is fixed at 0.025. All three values are in line with empirical macro estimates and represent values commonly used in the literature (see for instance Christiano and Eichenbaum (1992), Campbell (1994), Jermann (1998), King and Rebelo (1999), and Boldrin, Christiano and Fisher (2001)). Following earlier justification, the measure of managers, μ, is established at $\mu = .05$.

The remaining calibrated parameters principally concern the consumer-worker-shareholder's utility representation. In particular, following Hansen (1985) and many others, we choose his coefficient of the intertemporal elasticity of substitution in consumption to be $\eta_s = 1$ and the inverse of the Frisch elasticity of labor supply to be $\zeta = 0$. Time allocation studies (e.g., Ghez and Becker (1975) and Juster and Stafford (1991)) estimate that the average time devoted to market activities in the U.S. is equal to one third of discretionary time. Consequently, we choose B = 2.85 so that the steady state value of labor, $\overline{n},$ is equal to one-third of the time endowment. With these parameter choices, our model is directly analogous to the representative agent construct of Hansen (1985) – a long standing benchmark in the real business cycle literature.

Following standard practice, the log of the technology shock is assumed governed by the AR(1) process:

$$\lambda_t = \rho\lambda_{t-1} + \varepsilon_t$$

Several studies have found that this process is highly persistent (see Prescott (1986) for details). Accordingly, we choose the value of the persistence parameter, ρ, equal to 0.95. When the model economy is driven by the technology shocks alone, we select the volatility, σ_ε, to match the empirical standard deviation of departures from trend output in the U.S. data (1.81%). The i.i.d. sunspot shock, if present, is distributed N(0; σ_v). As with the shock to productivity, we choose the standard deviation σ_v to match the volatility of output, σ_y, observed in the data, a common practice in the literature on indeterminacy when the model's fluctuations result from sunspots shocks only.

Our focus is exclusively on executive incentive contracts of the form (12) in equilibrium. Following Jensen, Murphy and Wruck (2004), the salary component $A/\overline{c}^m$ is fixed equal to one half of the overall steady state managerial compensation. In Section 3.3 we present some sensitivity analysis with respect to changes in this parameter.

Commonly accepted values for the second group of parameters are unavailable in the literature. This subset of parameters includes the manager's elasticity of intertemporal substitution in consumption, η_m, the share of the aggregate wage bill in the manager's contract, δ, and the contract convexity parameter, θ. When the model economy incorporates technology shocks and the sunspot shocks simultaneously, there is no obvious way to estimate the individual variances of these shocks or their correlation. This parameter vector thus includes σ_ϵ, σ_v, and $\rho_{\epsilon v}$.[31]

Following Jermann (1998) and others, we choose the parameter values for the second group in a way that maximizes the model's ability to replicate certain business cycle moments. Let ϑ denote the vector of remaining model parameters to calibrate, $\vartheta = [\eta_m,\ \delta,\ \theta,\ \sigma_\varepsilon,\ \sigma_v,\ \rho_{\varepsilon v}]'$, and g_T denote the set of data moments to match. In our case, g_T includes the standard deviations of detrended output, total consumption, investment, and labor, and contemporaneous correlations of consumption and labor with output characteristic of the U.S. economy. We calibrate ϑ as the solution that minimizes the following criterion:

$$J(\vartheta) = [g_T - f(\vartheta)]'\Sigma^{-1}[g_T - f(\vartheta)]$$

where $f(\vartheta)$ is the vector of moments implied by the model for a given realization of ϑ, Σ is a weighting matrix and g_T the vector of the point estimates of the target moments, computed using the data. The matrix Σ is a diagonal matrix with the standard errors of the estimates in g_T on

[31] To our knowledge, there is no reliable procedure to estimate the properties of the sunspot shock process from the data. Farmer and Guo (1995) and Salyer and Sheffrin (1998) try to identify sunspot shocks from rational expectations residuals that are left unexplained by exogenous shocks to fundamentals. But this technique is sensitive to model's misspecification and cannot distinguish between actual sunspots and missing fundamentals. Lubik and Schorfheide (2004) develop an econometric technique to assess the quantitative importance of equilibrium indeterminacy in dynamic stochastic general equilibrium models, such as ours.

the main diagonal. This calibration procedure thus minimizes a weighted average of the moment deviations.

Using the full set of parameters chosen as per above, the model's optimal policy functions were computed in the manner described in Appendix D, and artificial time series generated accordingly. These series were then detrended using the Hodrick-Prescott filter, and the model's business cycle statistics computed from their detrended components. We evaluated the criterion $J(\vartheta)$ for the following regions of parameter values: $\eta_m \in [0.00,\ 1.00]$, $\delta \in [0.01,\ 1.00]$, $\theta \in [0.5,\ 10]$, $\sigma_\varepsilon \in [0.005,\ 0.05]$, $\sigma_v \in [0.005,\ 0.1]$, and $\rho_{\varepsilon v} \in [-1,\ 1]$ with a partition norm equal to .01. The choice of regions for the first three parameters guarantees that the Baseline Model supports multiple equilibria.[32] Table 1 summarizes the results of the full calibration procedure.

[Insert Table 1 about here]

Given the other parameterized and estimated values in Table 1, the GMM estimate of $\theta = 3$ is conservative as compared to the elasticity of the manager's compensation with respect to firm value as estimated by Gabaix and Landier (2008). In particular, for the baseline parameterization of Table 1, we need to raise the overall contract convexity to $\theta = 5.47$ to obtain a steady-state elasticity of 1.14, the Gabaix and Landier (2008) estimate.[33]

3.2 Quantitative Results of the Baseline Model

To calculate business cycle statistics, we computed averages of the relevant statistical quantities for 500 sample paths each of 200 periods length. Summarizing model performance in this way is customary in the business cycle literature. This procedure does, however, tend to mask the sort of extreme behavior that might be associated with sunspot equilibria.

[32] Note also that these regions include the parameter values that identify the optimal contract.

[33] The steady-state elasticity of the manager's compensation with respect to the firm's stock price, $\overline{\xi} = \left(\overline{\frac{\partial g^m}{\partial q^e}}\right) \cdot \frac{\overline{q^e}}{\overline{g^m}}$, relates to the contract convexity as follows. Since $\overline{q}^e = \frac{\overline{d}}{1-\beta}$, and $\left(\overline{\frac{\partial g^m}{\partial q^e}}\right) = \left(\overline{\frac{\partial g^m}{\partial d}}\right)\left(\overline{\frac{\partial d}{\partial q^e}}\right) = \overline{x}\,(1-\beta)$, we can express $\overline{\xi} = \overline{x}\,(1-\beta)\,\frac{\overline{d}}{(1-\beta)\overline{g}^m} = \frac{\overline{x}\overline{d}}{\overline{g}^m}$. Using the steady-state relationships detailed in footnote 22, we have $\overline{d} = \overline{y} - \overline{wn} - \overline{i} - \mu\overline{g}^m$ $= \alpha\overline{y} - \Omega\overline{k} - \mu\overline{g}^m = \left(\frac{\alpha\left(\beta^{-1}-1\right)}{\left(\beta^{-1}-1+\Omega\right)}\right)\overline{y} - \mu\overline{g}^m$, where $\overline{g}^m = A + \varphi\left(\delta\overline{wn} + \overline{d}\right)^{\gamma\theta}$. Also, $\overline{x} = \theta\varphi\left(\delta\overline{wn} + \overline{d}\right)^{\gamma(\theta-1)}$. As a result, $\overline{\xi} = \frac{\overline{x}\overline{d}}{\overline{g}^m} = \frac{\varphi\theta\left(\delta\overline{wn}+\overline{d}\right)^{\gamma(\theta-1)}\cdot\overline{d}}{A+\varphi\left(\delta\overline{wn}+\overline{d}\right)^{\gamma\theta}}$.

Table 2 considers several fundamental cases. In Panel B, we present business cycle statistics obtained from the version of the model parameterized as in Table 1 with technology shocks being the single source of exogenous uncertainty. Nevertheless, the response of macro aggregates to this fundamental shock is indeterminate since the model, as parameterized, results in multiple equilibrium solutions. Panel C summarizes the model in which technology and sunspot shocks are both present with the indicated volatilities. In Panel D only the i.i.d. sunspot shock drives business cycle fluctuations. Panel A contains statistics estimated from the U.S. data, where available, and Panel E shows business cycle statistics from the Hansen (1985) indivisible labor model – a long-standing benchmark in the real business cycle literature. It also describes the analogous delegated management economy under the optimal contract for the indicated parameter values.

[Insert Table 2 about here]

Panel B (technology shock alone) easily respects the most basic stylized facts of the business cycle: investment is more volatile than output which is in turn more volatile than shareholder (and aggregate) consumption. Hours and investment are somewhat too smooth, however. Their relative standard deviations in the model are 0.24 and 1.91 respectively compared to 0.95 and 2.93 in the data. The Hansen (1985) model is more successful in replicating these particular statistics, but the model with the delegated manager is able to match the relative standard deviation of consumer-worker-shareholder's (and total) consumption more successfully. In the Hansen (1985) model, consumption does not vary enough. This happens because the equilibrium wage rate is more variable in our model than in Hansen's. When fluctuations in the model result from the technology shock alone, the manager's consumption is very smooth, more so than the consumption of shareholders. The manager changes investment and hiring policies in response to these shocks very moderately because in this case he does not expect any extra return on capital (no sunspots). This version of the model also replicates contemporaneous correlations with output as well as the Hansen (1985) model.

In Panel C, we incorporate sunspot shocks in addition to technology shocks into the model. This is the Baseline case. Comparison of results from Panels B and C shows that with two shocks, the model with the delegated manager and the convex executive incentive contract is able to match the empirical relative standard deviations of employment, investment and shareholders' consumption reasonably closely. For all the major aggregates the same can be said of the cross-correlations with output. Being a convex function of the dividend, which is itself a highly variable residual series in this version of the model, managerial consumption volatility appropriately exceeds that of the consumer-worker-shareholders.

Comparing the volatilities presented in Panel C with those obtained from U.S. data (Panel A), it would appear that sunspot shocks, when introduced into this production model setting, are fully harmonious with technology shocks (question 1). With the addition of sunspot shocks the model is also largely consistent with the stylized business cycle facts (question 2). In fact, the case with two shocks arguably does a better job of replicating the data than does the seminal paradigm of Hansen (1985) (Panel E). In particular, consumption volatility much more closely matches the data. The negative correlation of managerial consumption with output simply reflects the like correlation of its dividend base. Thus the answer to the second question, posed in the beginning of this section is also in the affirmative: sunspots can enhance the ability of the model to replicate salient business cycle facts.

Sunspot shocks alone (Panel D), however, give rise to a number of data inconsistencies. First, hours and investment are excessively volatile. Note that the volatility of the sunspot shock must more than double in order to compensate for the absence of technological uncertainty (Panel C vs. Panel D), if the required output volatility is to be maintained. This fact is not entirely surprising since sunspot shocks do not affect output directly, and thus must induce large responses in hours and investment in order to replicate σ_y at the empirically observed $\sigma_y = 1.81\%$. As a result, σ_i and σ_n are high relative to their empirical counterparts. Dividends give rise to most of the variation in managerial compensation and these are countercyclical. As a result, managerial consumption is countercyclical as well. Being a residual after the wage bill and investment, the dividend, and thus managerial consumption, is also highly volatile.

The other major inconsistency is reflected in the negative correlation of consumer-worker-shareholder consumption with output. By implication, total consumption is negatively correlated with output as well. Sunspot equilibria, per se, seem to have manifestations that violate the notion of consumption as a normal good, at least in this case. Recall that a sunspot shock is essentially a rate of return on capital stock, and that a favorable sunspot shock induces very large procyclical responses in investment without output being itself simultaneously increased. In equilibrium, consumption must therefore be countercyclical. Note that for all three of the considered cases contract convexity is a relatively modest $\theta = 3$.

3.3 Robustness Checks with Respect to Changes in Key Parameters

Table 3 explores the consequences of greater contract convexity, a larger salary component, and higher managerial risk aversion in the context of the Baseline case of Table 2, Panel C. We leave all other parameters (including standard deviations of shocks) unchanged (relative to the baseline case) to illustrate better the effect of changes in the three key parameters for indeterminacy on the ability of the model to replicate basic business cycle facts. The Table 3 cases illustrate indeterminate equilibria all of which provide reasonable replications of the statistical summary of the U.S. business cycle. It reflects the fact that indeterminacy is robust to a wide class of contract parameters and risk aversion levels for the manager.

[Insert Table 3 about here]

In Panel B (higher contract convexity with $\theta = 5$), the relative volatilities of hours and investment are lower than in the baseline case (Panel A) while managerial consumption volatility is higher. The latter phenomenon goes hand in hand, ceteris paribus, with greater contract convexity. Since the only tools the manager has for dealing with this greater uncertainty are his firm's investment and hiring decisions, he alters them to reduce their reaction to shocks. Accordingly, this action leads to lower investment and hours volatility relative to the baseline case. Had the manager employed the same investment and hours policies as in the baseline case, his consumption volatility under the higher contract convexity would have substantially exceeded the value reported.

Panel C of Table 3 explores the consequences of altering the level of the salary component. For the underlying parameters of $\eta_m = 0.25$ and $\theta = 3$, sensitivity analysis reveals that a minimum value for the salary component share in the manager's compensation of $A/\overline{c}^m = 0.15$ appears to be necessary for indeterminate equilibria to arise. Relative to the baseline case, increasing the magnitude of the fixed salary component increases the overall volatility in the economy once indeterminate equilibria are achieved. The effect is more pronounced in the case of the relative standard deviations of hours and investment. The reason is as follows: increasing the weight of the fixed salary in the overall compensation package of the manager makes him effectively more risk tolerant and more willing to alter production plans in response to shocks, hence the increases in investment and hours volatility. It simultaneously reduces his own relative consumption volatility, since a higher fraction of his mean consumption is fixed relative to the baseline.[34]

Panel D in Table 3 concerns the consequences of the enhanced managerial elasticity of intertemporal substitution in consumption (i.e. managerial risk aversion in case of the utility function (1)). Sensitivity analysis shows that the model economy exhibits local indeterminacy if the manager's risk aversion coefficient does not exceed 0.49 with other parameters held at their baseline values.

As is evident, the influence of the increase in managerial risk aversion is very modest provided the equilibria are indeterminate. Indeed, for Panel D, not only are the stylized business cycle facts quite well replicated but the results seem relatively unaffected (relative to the benchmark) by the degree of managerial risk aversion (provided $\eta_m < 0.49$). Furthermore, in none of the cases presented in Table 3 is managerial consumption volatility particularly excessive, ranging only to roughly four times that of shareholder-workers.

In summary, the preceding cases inform us along a number of dimensions with regard to the three questions posed at the start of this section. First, the results from not only the baseline case with technology and sunspot shocks (Table 2, Panel C) but also many of the other cases suggest that these sources of uncertainty are fully compatible with one another, at least for this model framework. Our quantitative results suggest that one source of uncertainty can effectively be traded off against the other (with regard to the relative magnitudes of σ_ε and σ_v), a direct consequence being that a large increase in future sunspot volatility would not be inconsistent with past business cycle history as it is currently understood and measured.

With regard to our second question, standard business cycle models (e.g., Hansen, 1985) have difficulty replicating the relative volatility of hours. The addition of sunspot shocks appears to resolve this shortcoming.

[34] In the base case $\frac{A}{\bar{c}^m} = .5$; in the case of Panel C, $\frac{A}{\bar{c}^m} = .7$.

Finally, within the realm of the simple construct we provide, it does not appear that convex-contract-induced sunspot equilibria, alone, can replicate the stylized facts of the business cycle. Shareholder consumption in these cases is negatively correlated with output and hours and investment exhibit excessive volatility. Such is the response to the third question posed.

4 Optimal Incentive Contracts

While the family of contracts (12) is both plausible and conforms, broadly speaking, to observed executive compensation practice, its origin is ad hoc. The following Theorem however establishes that contracts of the form (12) lead to a first-best allocation between managers and shareholder-workers in our model, for particular (and plausible) parameter choices, thereby providing an ultimate justification for their consideration.

Theorem 2 *A compensation contract of the form (12) with* $A = 0$*, a particular value* $\varphi > 0$*,* $\delta = 1$*,* $\gamma = \eta_s/\eta_m > 0$ *and* $\theta = 1$ *is optimal, in the sense that it results in the optimal plan. If, in addition, the consumer-worker-shareholders are more risk-averse than the managers,* $\eta_s > \eta_m$*, then* $\gamma > 1$ *and this optimal contract provides managerial compensation that is convex in the sum of the aggregate wage bill and aggregate dividends.*

Proof. See Appendix B ■

This theorem states that a contract of the form

$$g^m(w_t n_t, d_t, d_t(f)) = \varphi (w_t n_t + d_t)^{\eta_s/\eta_m} + (d_t(f) - d_t) \quad (45)$$

satisfies all constraints and is consistent with the optimal equilibrium. Such a compensation function $g^m(\quad)$ for each manager implies an equilibrium outcome that aligns the manager's interests with those of the consumer-worker-shareholders who hire him, i.e., the manager chooses the same investment and labor functions as the shareholder-workers would choose if they themselves managed the firm. Since all firms are identical, $d_t(f) = d_t$ for all f, in equilibrium each manager's compensation reduces to

$$g^m(w_t n_t, d_t, d_t(f)) = \varphi (w_t n_t + d_t)^{\eta_s/\eta_m}.$$

Recall that in this economy, the consumer-worker-shareholders do not confront any moral-hazard problem vis-à-vis the manager, as they both have the same (full) information set. Yet, even in this context, a convex contract (where $\eta_s > \eta_m$) may be required to align, properly, the marginal utilities of the consumer-worker-shareholders and of managers in all states of the world:

$$(c_t^s)^{-\eta_s} = \Lambda_4 (c_t^m)^{-\eta_m}. \tag{46}$$

As shown in Appendix B, $\Lambda_4 > 0$ is the Lagrange multiplier on the manager's participation constraint (11). The particular value φ entering the optimal contract is chosen to satisfy the manager's participation constraint and is determined by the welfare weights assigned to the two agents in the Pareto formulation; it is given by $\varphi = \Lambda_4^{1/\eta_m}$. Using the agents' budget constraints (2) and (10), we note that in equilibrium $c_t^s = w_t n_t + d_t$ and $c_t^m = g_t^m$, which combined with (46) yields

$$g_t^m = c_t^m = \Lambda_4^{1/\eta_m} (c_t^s)^{\eta_s/\eta_m} = \Lambda_4^{1/\eta_m} (w_t n_t + d_t)^{\eta_s/\eta_m}.$$

The proposed compensation contract is thus socially optimal in equilibrium.

The basic intuition underlying Theorem 2 may be summarized as follows: in order for the delegated manager to select the investment and hiring plans preferred by the consumer-worker-shareholders, he must (i) be given an income stream with the same stochastic characteristics and (ii) he must be equally sensitive to these same income variations.[35] By making equilibrium compensation depend on $w_t n_t + d_t$, the first of these requirements is satisfied. By raising this quantity to the power η_s/η_m, the marginal utility of the manager is made proportional to that of the consumer-shareholder-worker.

For instance, if $0 \le \eta_m < \eta_s$, then the consumer-worker-shareholders ideally offer a convex compensation contract to the relatively risk-loving manager, in order to counteract the manager's

[35] With homogeneous utility and constant return to scale production, the manager will make the same investment decisions irrespective of the scale of his income stream.

weak concavity in preferences. In contrast, if $0 \leq \eta_s < \eta_m$, contract concavity effectively induces the manager to behave in a more risk-averse fashion. The last term in (45) in turn guarantees that the manager makes the optimal intertemporal decisions, with physical capital investment the same as in the optimal plan.

More generally, in dynamic stochastic general equilibrium models such as the one considered here, dividends (i.e., free cash flows) are countercyclical. This fact induces the risk-averse manager to smooth out the firm's investment series much more than the consumer-worker-shareholders find optimal. To do otherwise would force the manager into a circumstance of very low consumption during cyclical upturns when investment is typically high. The convexity of the contract overcomes the aforementioned disincentive and induces the manager to adopt a much more strongly pro-cyclical investment plan.

In the limiting case that the manager is of measure $\mu = 0$, another optimal contract belonging to the family of contracts (12) exists, in addition to the contract derived in Theorem 2. It is characterized by $\delta = 1$, $\gamma = 1$, $A = 0$, $\varphi = \Lambda_4^{1/\eta_m}$ but $\theta = \frac{1-\eta_s}{1-\eta_m}$ with either $0 < \eta_m, \eta_s < 1$ or $\eta_m > 1$, and $\eta_s > 1$, so that the compensation takes the form $g^m(w_t n_t, d_t, d_t(f)) = \varphi(w_t n_t + d_t(f))^{\frac{1-\eta_s}{1-\eta_m}}$ (see Appendix E).[36] Such an optimal contract is convex also in the firm's own dividend ($\theta > 1$) provided that $0 < \eta_s < \eta_m < 1$ or $1 < \eta_m < \eta_s$. While such a compensation contract induces the manager to undertake that hiring and investment decisions that the consumer-shareholder-worker would have chosen, it does not require that managers and consumer-shareholder-workers equate their marginal utilities given that the measure of managers is zero.

4.1 Deviations from Optimal Contract and Indeterminacy/Instability

The discussions in Section 2 refer to the general family of incentive contracts of the form (12), and argue that the general equilibrium can be indeterminate or explosive if the contract convexity, θ, is excessive or if the fixed payment, A, is sufficiently large. A natural question is how "close" an optimal contract would be to these regions of equilibrium indeterminacy or instability. As we now show, while the optimal incentive contract would result in a unique stable equilibrium, slightly more generous incentive contracts could easily result in undesirable outcomes.

[36] Appendix E is available online from the authors' websites.

As stated in Theorem 2, the optimal contract requires that $A = 0$, $\delta = 1$, $\gamma = \eta_s/\eta_m$, and $\theta = 1$. It is convex in the sum of the aggregate wage bill and aggregate dividends if $\eta_s > \eta_m$. While this optimal contract, indicated by a $*$ in Figures 1, 2, and 3 lies in the region of determinacy, we can easily obtain equilibrium indeterminacy or instability if the contract specifies larger values of the fixed payment A, or a higher convexity θ in the sum of aggregate wages and the individual firm's dividend, or a lower weight δ on the aggregate wage bill. The risk of obtaining such undesirable equilibria is especially pronounced if η_m is low, i.e., if the manager's elasticity of intertemporal substitution is high.

Similarly, the shaded regions in Figure 4 present the parameter combinations for which indeterminacy or instability will arise in the case that $\mu = 0$.[37] The boldfaced curves representing the optimal contracts discussed above, for the case $\mu = 0$. Notice that the indeterminacy/instability region does not intersect with either of the boldfaced curves representing the optimal contracts, nor would it for any other choice of η_s. While the optimal contract does not per se lead to equilibrium indeterminacy or instability, a slightly more generous compensation in the form of higher contract convexity could easily result in such bad outcomes.

Since we assume full information in this model, these outcomes could also arise in an extension of the model where consumer-worker-shareholders, who determine the contract form and its parameters, know their own coefficient of relative risk aversion but mistakenly over-estimate the manager's true degree of risk aversion. For example, suppose that the true η_s and η_m satisfy $\eta_s = \eta_m = 0.5$, so that the optimal contract parameters are $A = 0$ and $\theta = 1$. If the shareholders counterfactually estimate $\eta_m = 0.8$, then they will choose contract parameters $A = 0$ and $\theta = \frac{1-\eta_s}{1-\eta_m} = 2.5$. Relative to the true η_m which guides the manager's actions, this choice of θ leads to indeterminacy or explosive equilibria as $\psi = \eta_m - \frac{\theta-1}{\theta} = 0.5 - \frac{1.5}{2.5} < 0$.

The larger shaded region (comprising the dark and light gray regions) similarly represents the region of indeterminacy or instability, but assuming a positive fraction of the manager's compensation in the form of a fixed payment. Specifically, it assumes $A/\bar{c}^m = 0.5$. As Figure 4 makes clear, for any η_m, the minimal magnitude of θ necessary for indeterminacy is strictly less than in the $A/\bar{c}^m = 0$ case. Note also that for a wide range of managers' elasticity of intertemporal

[37]The dark region shows the parameter configurations which satisfy $\psi < \psi^*$, for $A = 0, \mu = 0$.

substitution, the choice of convexity θ in the optimal contract leads to indeterminacy or instability when $A/\bar{c}^m = 0.5$. A larger positive fixed payment thus allows indeterminacy to arise for a much larger set of parameter configurations. We are also reminded that the presence or absence of indeterminacy is related only to the manager and the terms of his contract: nowhere does η_s enter into the definition of the region of indeterminacy. This observation follows from the fact that the manager alone determines the firm's investment decision in the delegated management economy.

5 Conclusion

The message of this paper is clear and direct: when confronted with compensation contracts which are mildly convex to the firm's stock price or free cash flow, CEOs may well find it in their self-interest to adopt investment policies that lead to equilibrium indeterminacy or instability. As a result, the time path of the economy's macroeconomic aggregates, as well as the executives' compensation, at least with respect to volatility, may bear little association to fundamentals. In this sense, highly convex CEO compensation contracts may substitute for technological increasing returns, a typical requirement of the earlier indeterminacy literature. Within a standard dynamic macroeconomic setup, these results appear to hold for a wide class of model parameters, at least for the compensation contracts studied here.

These results suggest that the early twenty-first century explosion in the incentive compensation among financial firms may have unforeseen consequences. We are only now beginning to see what these consequences are.

A Appendix A: Proof of Theorem 1

To characterize the regions of determinacy, indeterminacy and instability, we rewrite the dynamic equations (39)–(40) in matrix form:

$$A \begin{bmatrix} E_t \hat{c}^m_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = B \begin{bmatrix} \hat{c}^m_t \\ \hat{k}_t \end{bmatrix} + C\lambda_t$$

where

$$A \equiv \begin{bmatrix} 1 & A_{12} \\ 0 & 1 \end{bmatrix}, \quad B \equiv \begin{bmatrix} 1 & 0 \\ B_{21} & B_{22} \end{bmatrix},$$

$$A_{12} = \psi^{-1} (1-\alpha)(1-\beta(1-\Omega)),$$

and B_{21} and B_{22} are given in (41)–(42). Note that $sign(A_{12}) = sign(\psi)$.

Given that $\hat{k}_t$ is a predetermined variable and $\hat{c}^m_t$ is a nonpredetermined variable, the system admits a single bounded solution if and only if the eigenvalues ϕ_1 and ϕ_2 of the matrix $M \equiv A^{-1}B$ satisfy $0 \leq |\phi_1| < 1 < |\phi_2|$. The equilibrium is indeterminate if $0 \leq |\phi_1| < 1$ and $0 \leq |\phi_2| < 1$. There exists no bounded solution (and so there exist only explosive solutions) if $1 < |\phi_1|$ and $1 < |\phi_2|$.

It will be useful to appeal to the following proposition:

Proposition 3 *(Proposition C1 of Woodford (2003, p. 670)). Both eigenvalues of a 2×2 matrix N lie outside the unit circle if and only if:*
either (Case I):

$$\det(N) > 1 \tag{A.1}$$

$$\det(N) - tr(N) + 1 > 0 \tag{A.2}$$

$$\det(N) + tr(N) + 1 > 0 \tag{A.3}$$

or (Case II):

$$\det(N) - tr(N) + 1 < 0 \tag{A.4}$$

$$\det(N) + tr(N) + 1 < 0. \tag{A.5}$$

A.1 Indeterminacy

The system has an indeterminate equilibrium if and only if $1 < |1/\phi_1|$ and $1 < |1/\phi_2|$, or equivalently if both eigenvalues of

$$M^{-1} = \begin{bmatrix} 1 & A_{12} \\ -\frac{B_{21}}{B_{22}} & \frac{1}{B_{22}}(1 - A_{12}B_{21}) \end{bmatrix}$$

lie outside the unit circle. Note that $\det(M^{-1}) = 1/B_{22}$ and $tr(M^{-1}) = 1 + (1 - A_{12}B_{21})/B_{22}$.

First suppose that $\det(M^{-1}) - tr(M^{-1}) + 1 = A_{12}\frac{B_{21}}{B_{22}} < 0$. Since $B_{22} > 0$, we must have $\det(M^{-1}) + tr(M^{-1}) + 1 = 2\left(\frac{B_{22}+1}{B_{22}}\right) - A_{12}\frac{B_{21}}{B_{22}} > 0$. So we cannot simultaneously satisfy both conditions (A.4)–(A.5) of Case II.

We thus have an indeterminate equilibrium if and only if all three conditions (A.1)–(A.3) of

Case I are satisfied, i.e., if and only if

$$\begin{aligned} 0 &< B_{22} < 1 \\ A_{12}\frac{B_{21}}{B_{22}} &> 0 \\ 2\left(\frac{B_{22}+1}{B_{22}}\right) - A_{12}\frac{B_{21}}{B_{22}} &> 0. \end{aligned}$$

Given that B_{22} is positive and increasing in δ, the first condition is satisfied for δ sufficiently small

$$\left(\beta^{-1} - (1-\Omega)\right)(1-\alpha)\,\delta + (1-\Omega)(1-\alpha) + \alpha\beta^{-1} < 1$$

or equivalently

$$\delta < \delta^* \equiv \frac{1-(1-\Omega)(1-\alpha)-\alpha\beta^{-1}}{\left(\beta^{-1}-(1-\Omega)\right)(1-\alpha)}. \tag{A.6}$$

Given $B_{22} > 0$ and $B_{21} < 0$, the second condition is satisfied if and only if $A_{12} < 0$, or equivalently if and only if

$$\psi < 0. \tag{A.7}$$

The third condition is in turn satisfied if and only if $2(B_{22}+1) > \psi^{-1}(1-\alpha)(1-\beta(1-\Omega))B_{21}$ or equivalently if and only if

$$\psi^{-1}\psi^* < 1 \tag{A.8}$$

where

$$\psi^* \equiv \frac{(1-\alpha)(1-\beta(1-\Omega))B_{21}}{2(B_{22}+1)} < 0.$$

If (A.7) holds, then (A.8) can be rewritten as $\psi < \psi^*$. It follows that conditions (A.7) and (A.8) jointly hold if and only if

$$\psi < \psi^* < 0. \tag{A.9}$$

To summarize, the model admits an indeterminate equilibrium if and only if (A.6) and (A.9) are jointly satisfied.

A.2 Instability

The system admits no bounded solution if and only if both eigenvalues of M lie outside the unit circle. Note that $\det(M) = B_{22}$ and $tr(M) = 1 + B_{22} - A_{12}B_{21}$.

First suppose that $\det(M) - tr(M) + 1 = A_{12}B_{21} < 0$. Since $B_{22} > 0$, we must have $\det(M) + tr(M) + 1 = 2(B_{22}+1) - A_{12}B_{21} > 0$. So we cannot simultaneously satisfy both conditions (A.4)–(A.5) of Case II. Thus we have explosive solutions if and only if all three conditions (A.1)–(A.3) of Case I are satisfied, that is:

$$\begin{aligned} B_{22} &> 1 \\ A_{12}B_{21} &> 0 \\ 2(B_{22}+1) - A_{12}B_{21} &> 0. \end{aligned}$$

The first condition is satisfied for δ sufficiently large

$$\delta > \delta^*. \tag{A.10}$$

Given $B_{21} < 0$, the second condition is satisfied if and only if $A_{12} < 0$, or equivalently if and only if (A.7) holds. The third condition is in turn satisfied if and only if (A.8) holds. The second and third conditions thus jointly hold if and only if (A.9) is satisfied. To summarize, the model admits no bounded solution if and only if (A.9) and (A.10) are jointly satisfied.

A.3 Determinacy

The system admits a unique bounded solution if and only if the equilibrium is neither indeterminate nor unstable, that is, if and only if $\psi \geq \psi^*$. ■

B Appendix B: Proof of Theorem 2

To prove Theorem 2 we need to show that in the case that the coefficients of a contract (12) take the values stated in the Theorem 2, the resulting equilibrium conditions — given by (4)–(5), (19)–(28), and (11) holding with equality — imply the same equilibrium as the optimal plan which specifies the optimal allocation of consumption, employment, investment, and payments at all dates and in all states of the world. We proceed with the characterization of the optimal plan.

B.1 Optimal Plan

The optimal plan can be characterized by maximizing the consumer-shareholder-worker's utility (1) subject to the constraints (2), (6)–(8), (10) at all dates and in all states, and the manager's participation constraint (11) in all states. Noting that (10) and (11) must hold with equality in an optimal equilibrium, and combining (6) and (7) with (8), we can write the Lagrangian

$$\begin{aligned}
\mathcal{L} \;=\; & E_0 \Bigg\{ \sum_{t=0}^{\infty} \beta^t \Bigg(\frac{(c_t^s)^{1-\eta_s}}{1-\eta_s} - B \frac{(n_t^s)^{1+\zeta}}{1+\zeta} \\
& + \Lambda_{1t} \left[\int_0^1 \left(q_t^e(f) + d_t(f)\right) z_t^e(f)\, df + w_t n_t^s + b_t^s - c_t^s - \int_0^1 q_t^e(f)\, z_{t+1}^e(f)\, df - q_t^b b_{t+1}^s \right] \\
& + \int_0^1 \Lambda_{2t}(f) \left[(k_t(f))^{\alpha} (n_t(f))^{1-\alpha} e^{\lambda_t} - w_t n_t(f) - k_{t+1}(f) + (1-\Omega) k_t(f) - \mu g_t^m(f) - d_t(f) \right] df \\
& + \int_0^1 \mu \Lambda_{3t}(f) \left[g_t^m(f) - c_t^m(f) \right] df \\
& + \int_0^1 \mu \Lambda_4(f) \left[\frac{(c_t^m(f))^{1-\eta_m}}{1-\eta_m} - \bar{u}^m \right] df \Bigg) \Bigg\},
\end{aligned} \tag{B.1}$$

where $\{\Lambda_{1t}, \Lambda_{2t}(f), \mu\Lambda_{3t}(f)\}_{t=0}^{\infty}$ are the Lagrange multipliers associated, respectively, to the constraints (2), (8), and (10), and $\mu\Lambda_4(f) \geq 0$ is the multiplier associated with constraint (11).

The necessary and sufficient first-order conditions with respect to $c_t^s, n_t^s, z_{t+1}^e(f), b_{t+1}^s, k_{t+1}(f),$

$n_t(f), c_t^m(f), d_t(f)$ and the optimal compensation $g_t^m(f)$ imply

$$\begin{aligned}
\Lambda_{1t} &= (c_t^s)^{-\eta_s} && \text{(B.2)}\\
\Lambda_{1t} w_t &= B(n_t^s)^{\zeta} && \text{(B.3)}\\
q_t^e(f) &= E_t\left[\beta \frac{\Lambda_{1t+1}}{\Lambda_{1t}} \left(q_{t+1}^e(f) + d_{t+1}(f)\right)\right] && \text{(B.4)}\\
q_t^b &= E_t\left[\beta \frac{\Lambda_{1t+1}}{\Lambda_{1t}}\right] && \text{(B.5)}\\
\Lambda_{2t}(f) &= E_t\left\{\beta \Lambda_{2t+1}(f)\left[1 - \Omega + \alpha y_{t+1}(f)/k_{t+1}(f)\right]\right\} && \text{(B.6)}\\
w_t &= (1-\alpha) y_t(f)/n_t(f) && \text{(B.7)}\\
(c_t^m(f))^{-\eta_m} &= \frac{\Lambda_{3t}(f)}{\Lambda_4(f)} && \text{(B.8)}\\
\Lambda_{2t}(f) &= \Lambda_{1t} z_t^e(f) && \text{(B.9)}\\
\Lambda_{2t}(f) &= \Lambda_{3t}(f). && \text{(B.10)}
\end{aligned}$$

The optimal equilibrium is a set of processes $\{c_t^s, c_t^m(f), n_t^s, n_t(f), b_t^s, z_t^e(f), i_t(f), y_t(f), k_t(f), w_t, q_t^b, q_t^e(f), d_t(f), g_t^m(f)\}$ and Lagrange multipliers $\{\Lambda_{1t}, \Lambda_{2t}(f), \Lambda_{3t}(f)\}$, and $\Lambda_4(f)$ such that:

1. The first-order conditions (B.2)–(B.10) are satisfied together with the constraints (2), (6)–(8), (10)–(11), all holding with equality, and the transversality conditions: $\lim_{t\to\infty} \beta^t (c_t^m(f))^{-\eta_m} k_{t+1}(f) = 0$, for any given initial k_0 common to all firms.

2. The labor, goods and capital markets clear: $n_t^s = n_t$; $y_t = c_t^s + \mu c_t^m + i_t$ where, as before, we denote by n_t, y_t, c_t^m, i_t the aggregates over all firms, i.e., $n_t \equiv \int_0^1 n_t(f)\,df$, and so on; investors hold all outstanding equity shares normalized to one for each firm, $z_t^e(f) = 1$, and all other assets (one period bonds) are in zero net supply, $b_t^s = 0$.

Since all firms and hence all managers face the same constraints and solve the same problem, they all make the same decisions. We may thus without loss of generality characterize the optimal equilibrium in terms of aggregate variables only, replacing all variables indexed by f with their aggregate counterpart (e.g., $y_t(f) = y_t \equiv \int_0^1 y_t(f)\,df$ and so on for all f). It is noteworthy that the equilibrium just described is the Pareto-efficient or first-best equilibrium.

It follows from (B.2), (B.9) and (B.10) that in equilibrium

$$\Lambda_{1t} = \Lambda_{2t} = \Lambda_{3t} = (c_t^s)^{-\eta_s}.$$

This "multiple equality," together with (B.8), implies, furthermore, that the marginal utilities of consumption of the consumer-shareholder-worker and of the manager must be proportional in the optimal equilibrium, i.e.,

$$(c_t^m)^{-\eta_m} = \Lambda_4^{-1} (c_t^s)^{-\eta_s} \tag{B.11}$$

at all dates and in all states, where Λ_4 depends, among other factors, on the manager's reservation utility $\bar{u}^m$. Note that $\Lambda_4 > 0$ since the constraint (11) holds with equality in equilibrium. Equation (B.11) guarantees perfect risk sharing between the consumer-worker-shareholder and the manager.

The remaining equilibrium conditions can be written as follows:

$$
\begin{aligned}
\left(c_t^s\right)^{-\eta_s} w_t &= B n_t^{\zeta} && \text{(B.12)}\\
q_t^e &= E_t\left[\beta\left(c_{t+1}^s/c_t^s\right)^{-\eta_s}\left(q_{t+1}^e+d_{t+1}\right)\right] && \text{(B.13)}\\
q_t^b &= E_t\left[\beta\left(c_{t+1}^s/c_t^s\right)^{-\eta_s}\right] && \text{(B.14)}\\
\left(c_t^s\right)^{-\eta_s} &= E_t\left[\beta\left(c_{t+1}^s\right)^{-\eta_s} r_{t+1}\right] && \text{(B.15)}\\
r_t &\equiv \alpha\left(y_t/k_t\right)+1-\Omega && \text{(B.16)}\\
w_t &= (1-\alpha)\, y_t/n_t && \text{(B.17)}\\
c_t^s &= w_t n_t + d_t && \text{(B.18)}\\
c_t^m &= g_t^m && \text{(B.19)}\\
d_t &= y_t - w_t n_t - i_t - \mu g_t^m && \text{(B.20)}\\
y_t &= k_t^{\alpha} n_t^{1-\alpha} e^{\lambda_t} && \text{(B.21)}\\
k_{t+1} &= (1-\Omega)\, k_t + i_t, \qquad k_0 \text{ given} && \text{(B.22)}\\
\frac{\bar{u}^m}{1-\beta} &= E_0\left[\sum_{t=0}^{\infty}\beta^t \frac{\left(c_t^m\right)^{1-\eta_m}}{1-\eta_m}\right]. && \text{(B.23)}
\end{aligned}
$$

B.2 Equivalence of Decentralized Equilibrium and Optimal Plan

We now show that when the contract coefficients satisfy the restrictions stated in Theorem 2 and φ is set equal to $\Lambda_4^{1/\eta_m} = \bar{c}^m / \left(\bar{c}^s\right)^{\eta_s/\eta_m}$, the equilibrium implied by conditions (4)–(5), (19)–(28), and (11) holding with equality is equivalent to the optimal equilibrium implied by (B.11)–(B.23).

Under such parameter restrictions, equation (20) reduces to

$$
c_t^m = g_t^m = \Lambda_4^{1/\eta_m}\left(w_t n_t + d_t\right)^{\eta_s/\eta_m} \qquad \text{(B.24)}
$$

so that the consumption of the shareholder and the manager, (21), (B.24), satisfy the optimal risk sharing condition (B.11), as well as (B.18) and (B.19). The variable x_t, given in (19), reduces to the constant $x_t = \varphi = \Lambda_4^{1/\eta_m}$ so that (28) reduces to

$$
\left(c_t^m\right)^{-\eta_m} = \beta E_t\left[\left(c_{t+1}^m\right)^{-\eta_m} r_{t+1}\right]. \qquad \text{(B.25)}
$$

This, together with (B.11) just established, yields the shareholder's optimal intertemporal allocation of consumption (B.15). All remaining conditions (4)–(5), (11), (22)–(27) are identical to the remaining equations characterizing the optimal plan, (B.12)–(B.14), (B.16), (B.17), (B.20)–(B.23).

Conversely, combining (B.11) and (B.18) implies (B.24), which is equivalent to (20) under the restrictions stated in Theorem 2. Combining (B.11), (B.15) and (B.24) yields in turn (B.25), which is equivalent to (28) under the restrictions considered. All remaining conditions characterizing the optimal plan (B.12)–(B.14), (B.16), (B.17), (B.20)–(B.23) are equivalent to (4)–(5), (11), (21)–(27).

It follows that for the contract stated in Theorem 2, the equilibrium implied by conditions (4)–(5), (19)–(28), and (11) holding with equality is equivalent to the optimal equilibrium.

References

[1] Benhabib, Jess and Roger E. Farmer, 1996, "Indeterminacy and Sector Specific Externalities," *Journal of Monetary Economics*, 37, 397-419.

[2] Benhabib, Jess, Qinglai Meng and Kazuo Nishimura, 2000, "Indeterminacy under Constant Returns to Scale in Multisector Economies," *Econometrica*, 68(6), 1541-1548.

[3] Bilbiie, Florin O., 2008, "Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic," *Journal of Economic Theory*, 140(1), 162-196.

[4] Boldrin, Michele, Lawrence J. Christiano, and Jonas Fisher, 2001, "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review*, 91, 149-166.

[5] Campbell, John, 1994, "Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model," *Journal of Monetary Economics*, 33, 463-506.

[6] Christiano, Lawrence J. and Martin Eichenbaum, 1992, "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations," *American Economic Review*, 82, 430-50.

[7] Clarida, Richard, Jordi Galí, and Mark Gertler, 2000, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics,* 115(1), 147-80.

[8] Cooley, Thomas F. and Edward C. Prescott, 1995, "Economic Growth and Business Cycles" in Thomas F. Cooley (ed.) *Frontiers of Business Cycle Research*, 1-38.

[9] Danthine, Jean-Pierre and John B. Donaldson, 2008a, "Executive Compensation: A General Equilibrium Perspective," working paper, The Swiss Finance Institute.

[10] ——— and ———, 2008b, "Executive Compensation and Stock Options: An Inconvenient Truth," working paper, The Swiss Finance Institute.

[11] Farmer, Roger E.A. and Jang-Ting Guo, 1995, "The Econometrics of Indeterminacy: An Applied Study," Carnegie-Rochester Conference Series on Public Policy, 43, 225-271.

[12] Frydman, C. and D. Jenter, 2010, "CEO Compensation," *Annual Review of Financial Economics* 2(1), 75-102.

[13] Gabaix, Xavier and Augustin Landier, 2008, "Why Has CEO Pay Increased so Much?" *Quarterly Journal of Economics,* 123(1), 49-100.

[14] Ghez, Gilbert and Gary S. Becker, 1975, *The Allocation of Time and Goods over the Life Cycle*, Columbia University Press.

[15] Hall, Brian J. and Kevin J. Murphy, 2002, "Stock Options for Undiversified Executives," *Journal of Accounting and Economics*, 33, 3-42.

[16] Hansen, Gary D., 1985, "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16, 309-327.

[17] Jaimovich, Nir, 2007, "Firm Dynamics and Markup Variations: Implications for Multiple Equilibria and Endogenous Economic Fluctuations," *Journal of Economic Theory*, 137(1), 300-325.

[18] Jensen, Michael C. and Kevin J. Murphy, 1990, "Performance Pay and Top-Management Incentives," *Journal of Political Economy*, 98, 225-264.

[19] Jensen, Michael C., Kevin J. Murphy. and Eric G. Wruck, 2004,"Remuneration: Where We've Been, How We Got to Here, What Are the Problems, and How to Fix Them," Harvard NOM Working Paper No. 04-28.

[20] Jermann, Urban, 1998, "Asset Pricing in Production Economies," *Journal of Monetary Economics*, 41, 257-275.

[21] Juster, F. Thomas and Frank P. Stafford, 1991, 'The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," *Journal of Economic Literature*, 29, 471-522.

[22] King, Robert G., Charles I. Plosser, and Sergio T. Rebelo, 1988, "Production, Growth and Business Cycles," *Journal of Monetary Economics*, 21, 195-232.

[23] King, Robert G. and Sergio T. Rebelo, 1999, "Resuscitating Real Business Cycles," in J.B. Taylor and M. Woodford (eds.), *The Handbook of Macroeconomics*, 1 (14), 927—1036.

[24] Lubik, Thomas A. and Frank Schorfheide, 2003, "Computing Sunspot Equilibria in Linear Rational Expectations Models," *Journal of Economic Dynamics and Control*, 28(2), 273-285.

[25] Murphy, Kevin J., 1999, "Executive Compensation," in Orley Ashenfelter and David Card (eds.), *Handbook of Labor Economics*, Vol. 3B, North Holland, 2485-2567.

[26] Perli, Roberto, 1998, "Indeterminacy, Home Production, and the Business Cycle: A Calibrated Analysis," *Journal of Monetary Economics*, 41(1), 105-125.

[27] Prescott, Edward C., 1986, "Theory Ahead of Business-Cycle Measurement," Carnegie-Rochester Conference Series in Public Policy 25, 11-44.

[28] Salyer, Kevin D. and Steven M. Sheffrin, 1998, "Spotting Sunspots: Some Evidence in Support of Models with Self-Fulfilling Prophecies," *Journal of Monetary Economics*, 42(3), 511-523.

[29] Shleifer Andrei and Robert Vishny, 1997, "A Survey of Corporate Governance," *Journal of Finance* 52, 737-783.

[30] Sims, Christopher A., 2000, "Solving Linear Rational Expectations Models," manuscript, Princeton University.

[31] Stock, James H. and Mark Watson, 1999, "Business Cycle Fluctuations in U.S. Macroeconomic Time Series," in J.B. Taylor and M. Woodford (eds.), *The Handbook of Macroeconomics,* 1(1), 3-64, North-Holland.

[32] Wen, Yi, 1998, "Capacity Utilization under Increasing Returns to Scale," *Journal of Economic Theory*, 81, 7-36.

[33] Woodford, Michael, 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, Princeton University Press.

Table 1 – Parameter Choices: Baseline Model

Parameter	Symbol	Value
Capital share of output	α	0.36
Subjective discount factor	β	0.99
Capital depreciation rate	Ω	0.025
Shareholders' elasticity of intertemporal substitution in consumption	η_s	1
Disutility of labor parameter	B	2.86
Inverse of the Frisch elasticity of labor supply	ζ	0
Persistence of the technology process	ρ	0.95
Measure of managers	μ	.05
Fraction of the salary component in the managerial contract	$A/\overline{c}^m$	0.5
Manager's elasticity of intertemporal substitution in consumption	η_m	0.25
Percentage of the aggregate wage in the managerial contract	δ	0.2
Convexity of the managerial contract	θ	3
Weighting on the variable compensation component	φ	4.894
Standard deviation of the technology shock	σ_ϵ	0.0107
Standard deviation of the sunspot shock	σ_v	0.0570
Correlation coefficient between the technology shock and the sunspot	$\rho_{\epsilon v}$	0
Relative compensation weight on the wage bill	γ	1

Table 2 — Business Cycle Statistics[(i)]

Statistic	Panel A U.S. data[(ii)]	Panel B[(i)] Technology shocks only	Panel C Technology and sunspot shocks. (Baseline)	Panel D Sunspot shocks only	Panel E Hansen
I. Standard Deviations (in percent)					
σ_y	1.81	1.81	1.81	1.81	1.81
σ_k/σ_y	0.35	0.18	0.30	0.56	0.26
σ_n/σ_y	.95	0.24	0.77	1.56	0.77
σ_i/σ_y	2.93	1.91	3.24	5.83	3.01
σ_{c^s}/σ_y	0.75	0.76	0.73	0.61	0.29
σ_{c^m}/σ_y	NA	0.13	3.98	8.37	NA
σ_ε	NA	1.22	1.07	NA	0.0073
σ_ν	NA	NA	5.70	11.90	NA
II. Contemporaneous Correlations with Output					
$\rho_{k,y}$	0.06	0.20	0.24	0.38	0.05
$\rho_{n,y}$	0.88	0.99	0.64	0.98	0.98
$\rho_{i,y}$	0.80	1.00	0.86	0.99	0.99
$\rho_{c^s,y}$	0.88	1.00	0.64	-0.86	0.87
$\rho_{c^m,y}$	NA	-0.03	-0.46	-0.97	NA

[(i)] All parameters, where applicable, are as in Table 1

[(ii)] Data from Stock and Watson (1999) as reported in King and Rebelo (1999).

Table 3 — Simulations with Alternative Parameterizations of the Managerial Contract[(i), (ii)]

	Panel A Baseline	Panel B High contract convexity $\theta = 5$	Panel C Large salary component $A/\bar{c}^m = 0.7$	Panel D High manager's risk aversion $\eta_m = 0.45$
		Standard Deviations		
σ_y	1.81	1.68	2.04	1.82
σ_k/σ_y	0.30	0.24	0.37	0.31
σ_n/σ_y	0.77	0.54	1.00	0.77
σ_i/σ_y	3.24	2.63	3.90	3.23
σ_{c^s}/σ_y	0.73	0.77	0.72	0.73
σ_{c^m}/σ_y	3.98	4.30	3.55	3.97
σ_ε	1.07	1.07	1.07	1.07
σ_ν	5.70	5.70	5.70	5.70
		Contemporaneous Correlations with Output		
$\rho_{k,y}$	0.24	0.23	0.27	0.24
$\rho_{n,y}$	0.69	0.68	0.74	0.69
$\rho_{i,y}$	0.86	0.87	0.87	0.86
$\rho_{c^s,y}$	0.64	0.85	0.35	0.64
$\rho_{c^m,y}$	-0.46	-0.30	-0.62	-0.46

[(i)] Except for those indicated, all parameters are as in Table 1.

[(ii)] These parameter choices lead to steady states which differ from that of the Benchmark.

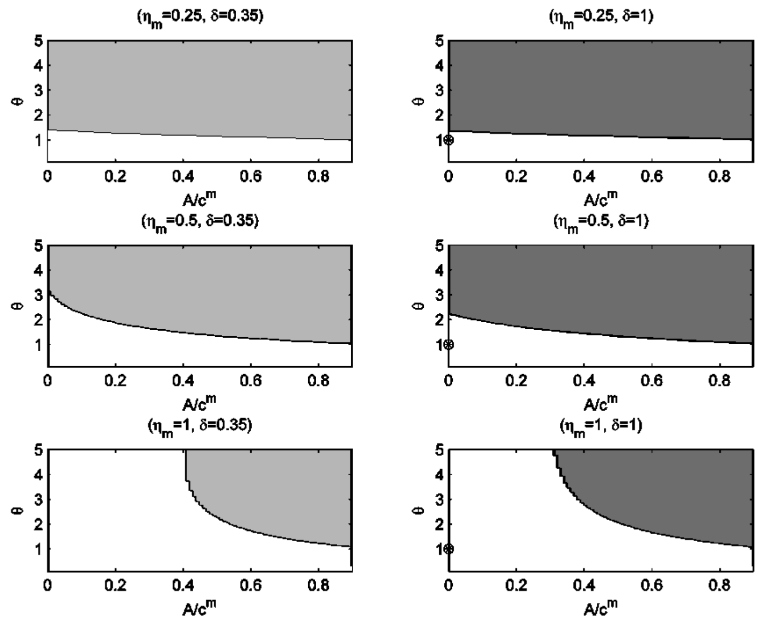

Figure 1: Regions of determinacy (white), indeterminacy (light gray) and instability (dark gray) in the case of fixed labor supply (Frisch elasticity $\zeta^{-1} \mapsto 0$). In all cases $\beta = .99$, $\alpha = .36$, $\mu = .05$, $\gamma = 1$, $\Omega = .025$ (these define a critical value of $\delta^* = .55$), and $\eta_s = 1$. Departures from optimal contract (Section 4) design include the possibility that $\delta < 1$, $A > 0$, and $\theta \neq 1$.

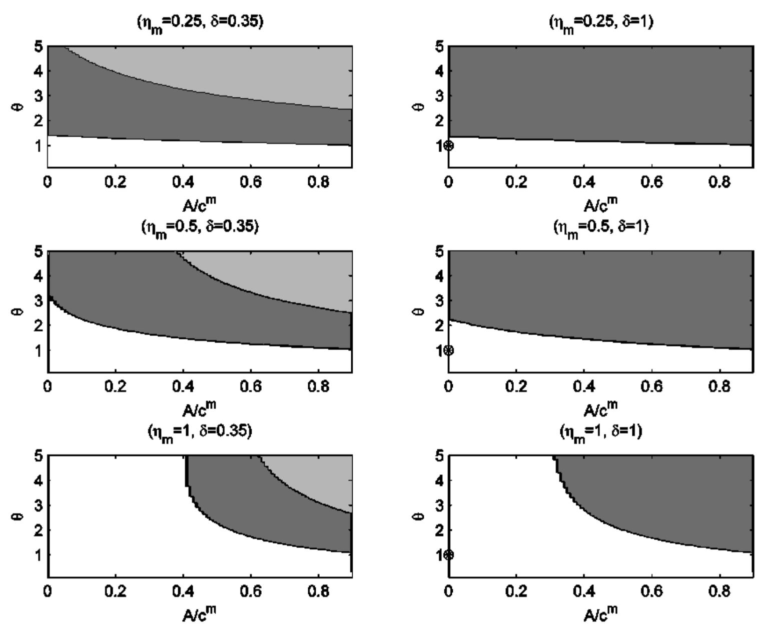

Figure 2: Regions of determinacy (white), indeterminacy (light gray) and instability (dark gray) in the case of variable labor supply (Frisch elasticity $\zeta^{-1} = 0.5$). All other parameter values are as in Figure 1, as well as the admissible departures from optimal contract design (Section 4).

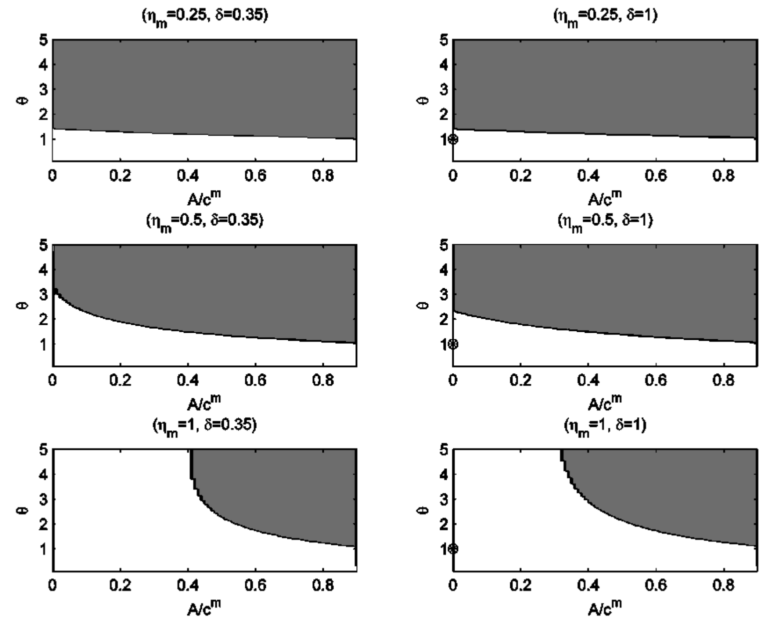

Figure 3: Regions of determinacy (white), indeterminacy (light gray) and instability (dark gray) with Hansen (1985) labor supply. (Frisch elasticity $\zeta^{-1} \rightarrow \infty$). All other parameter values and admissible departures from optimal contract design are as in Figures 1 and 2.

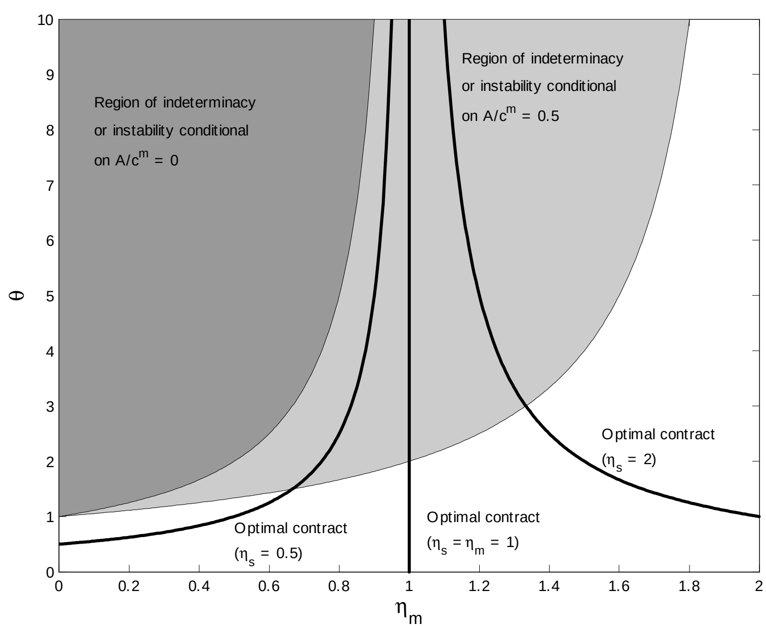

Figure 4: Regions of indeterminacy and instability, and optimal contract parameter combinations; $\mu = 0$ is assumed .

ADDITIONAL APPENDICES NOT FOR PUBLICATION

Some Unpleasant General Equilibrium Implications of Executive Incentive Compensation Contracts

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December 2011

The appendices contained in this document show the following:

- Appendix C: shows that there exists a set of parameters that satisfy the manager's participation constraint in addition to all equilibrium conditions.
- Appendix D: presents the numerical strategy.
- Appendix E: derives an alternative optimal contract in the case that $\mu = 0$.
- Appendix F: derives the generalized model with manager participating on financial markets and shows that the results reported in the paper are reinforced in this case.

C Appendix C: Satisfaction of Manager's Participation Constraint

We show that there exists a set of model structural parameters and contract parameters that satisfy the manager's participation constraint in addition to all equilibrium conditions. Let the model structural parameters $\{\alpha, \beta, \Omega, \eta_s, \eta_m, \mu, B, \zeta\}$ be given, and let $P = \{A, \varphi, \theta, \gamma, \delta\}$ be the set of contract parameters. We show there exists a set of contract parameters P which, in conjunction with the structural parameters, satisfy the following criteria:

(a) There exists a steady state $\left\{\overline{k}, \overline{n}, \overline{c}^m, \overline{c}^s\right\}$ that satisfies the model structural equations. That is, based on steady-state relationships mentioned in footnote 22, $\left\{\overline{k}, \overline{n}, \overline{c}^m, \overline{c}^s\right\}$ satisfy:

$$\overline{k} = \Gamma\overline{n} \text{ where } \Gamma = \left[\left(\beta^{-1} - 1 + \Omega\right)/\alpha\right]^{\frac{1}{\alpha-1}} \tag{C.1}$$

$$B\overline{n}^{\zeta} = (\overline{c}^s)^{-\eta_s}(1-\alpha)\Gamma^{\alpha} \tag{C.2}$$

$$\overline{c}^s = \Gamma^{\alpha}\overline{n} - \Omega\Gamma\overline{n} - \mu\overline{c}^m \tag{C.3}$$

$$\bar{d}/\bar{k} = \alpha\Gamma^{\alpha-1} - \Omega - \mu\bar{g}^m/\bar{k} = \beta^{-1} - 1 - \mu\bar{g}^m/\bar{k}, \tag{C.4}$$

$$\overline{c}^m = A + \varphi\left[\delta(1-\alpha)\Gamma^{\alpha}\overline{n} + \alpha\Gamma^{\alpha}\overline{n} - \Omega\overline{k} - \mu\overline{c}^m\right]^{\gamma\theta} \tag{C.5}$$

(b) The indeterminacy criterion of Theorem 1 is satisfied; i.e.,

$$\delta < \delta^* \equiv 1 - \frac{\beta^{-1} - 1}{\left(\beta^{-1} - 1 + \Omega\right)(1-\alpha)} < 1 \tag{C.6}$$

$$\psi < \psi^*, \tag{C.7}$$

where ψ and ψ^* are defined as in (38) and (44), that is

$$\psi \equiv \eta_m - \frac{\theta - 1}{\theta} \frac{1}{(1 - A/\bar{c}^m)(1 + \mu\bar{x})}, \qquad \psi^* \equiv \frac{(1-\alpha)(1 - \beta(1-\Omega)) B_{21}}{2(B_{22} + 1)} < 0,$$

and B_{21}, B_{22} are defined in (41)–(42), along with ω.

(c) The steady state incentive compatibility constraint is satisfied; i.e.,

$$\frac{(\bar{c}^m)^{1-\eta_m}}{1-\eta_m} = \frac{(\bar{c}^s)^{1-\eta_s}}{1-\eta_s} - B\frac{\bar{n}^{1+\zeta}}{1+\zeta}. \tag{C.8}$$

Consider first the case where $\mu = 0$. In this case, $\bar{k}, \bar{n}, \bar{c}^s, \bar{d}$ are exclusively determined by (C.1)–(C.4), and $\bar{c}^m$ is determined by (C.8). For appropriate parameterization of the kind used in any of our calibrations, $\bar{k}, \bar{n}, \bar{c}^m, \bar{c}^s, \bar{d}$ are all strictly positive and exist. It remains to show that there exist $A, \delta, \gamma, \theta$ and φ such that (C.5)–(C.7) are satisfied. Pick $\left\{\hat{\alpha}, \hat{\beta}, \hat{\Omega}, \hat{\eta}_s, \hat{\eta}_m, \hat{B}, \hat{\zeta}\right\}$ and $\mu = 0$ such that (C.1)–(C.4), and (C.8) yield strictly positive values of $\bar{k}, \bar{n}, \bar{c}^m, \bar{c}^s, \bar{d}$ and select some $\hat{\delta}$ satisfying $0 < \hat{\delta} < \hat{\delta}^*$, were $\hat{\delta}^*$ is defined as in (C.6). It follows from the definition of ω and (C.4) that $\omega = \frac{1-\alpha}{1-\alpha+\left(\bar{d}/\bar{k}\right)\Gamma^{1-\alpha}} = \frac{(1-\alpha)(1-\beta+\Omega\beta)}{1-\beta+\Omega\beta(1-\alpha)} \in (0,1)$. Equation (C.7) can then be rewritten as

$$\psi = \hat{\eta}_m - \frac{\theta - 1}{\theta} \frac{1}{1 - A/\bar{c}^m} < -\frac{(1-\hat{\alpha})\left(1 - \hat{\beta}\left(1 - \hat{\Omega}\right)\right)\left(\hat{\beta}^{-1} - 1\right)}{2(B_{22}+1)(1-\hat{\omega})\frac{\gamma\theta(1-A/\bar{c}^m)}{\delta\hat{\omega}+1-\hat{\omega}}} < 0.$$

This holds if and only if

$$\hat{\eta}_m \theta (1 - A/\bar{c}^m) - (\theta - 1) < -\frac{(1-\hat{\alpha})\left(1 - \hat{\beta}\left(1 - \hat{\Omega}\right)\right)\left(\hat{\beta}^{-1} - 1\right)}{2(B_{22}+1)(1-\hat{\omega})\frac{\gamma}{\delta\hat{\omega}+1-\hat{\omega}}} < 0$$

or equivalently if

$$\theta\left(1 - \hat{\eta}_m (1 - A/\bar{c}^m)\right) > 1 + \frac{(1-\hat{\alpha})\left(1 - \hat{\beta}\left(1 - \hat{\Omega}\right)\right)\left(\hat{\beta}^{-1} - 1\right)}{2(B_{22}+1)(1-\hat{\omega})\frac{\gamma}{\delta\hat{\omega}+1-\hat{\omega}}} > 0 \tag{C.9}$$

These last inequalities are satisfied for θ large enough and A close enough to $\bar{c}^m$, provided that $\gamma > 0$. It follows that there exists a non-empty connected set of contract parameters $P\left(\hat{\delta}; \hat{\alpha}, \hat{\beta}, \hat{\Omega}, \hat{\eta}_s, \hat{\eta}_m, \hat{B}, \hat{\zeta}\right)$ since $\widehat{\theta}$ can be chosen large enough and $A(\widehat{\theta})$ simultaneously close enough to $\bar{c}^m$ that (C.9) is satisfied with $0 < \hat{\varphi}$ subsequently selected so that (C.5) holds. The relevant set of parameters is thus $P = \bigcup_{0<\hat{\delta}<\delta^*} P\left(\hat{\delta}; \hat{\alpha}, \hat{\beta}, \hat{\Omega}, \hat{\eta}_s, \hat{\eta}_m, \hat{B}, \hat{\zeta}\right)$ for a given $\left(\hat{\alpha}, \hat{\beta}, \hat{\Omega}, \hat{\eta}_s, \hat{\eta}_m, \hat{B}, \hat{\zeta}\right)$.

Now suppose $\mu > 0$, but small. Note that the presence of μ affects only equations (C.3), (C.4) and (C.5) and the quantities B_{21} and ψ (via the $1 + \mu\hat{x}$ term as well as $\hat{x}$ itself), and does so in a continuous fashion. By continuity, there exists an open interval $(0, \mu^*)$ such that for any $\mu \in (0, \mu^*)$ the same equations and inequalities (C.1)–(C.8) must be satisfied.

D Appendix D: Numerical Strategy

The linearized rational expectations model presented in Section 3, can be rewritten in the canonical form:

$$\Gamma_0(\vartheta) s_t = \Gamma_1(\vartheta) s_{t-1} + \Psi(\vartheta) \varepsilon_t + \Pi(\vartheta) \tau_t \tag{D.1}$$

where the model parameters are collected in the vector $\vartheta = [\beta, \eta_s, B, \eta_m, \mu, \theta, \varphi, \delta, A, \alpha, \Omega, \rho, \sigma]$, with ρ and σ denoting, respectively, the persistence and the standard deviation of the exogenous shock, and s_t representing the vector of the model's endogenous variables:

$$s_t = \left[\hat{c}_t^s, \hat{c}_t^m, \hat{k}_t, \hat{n}_t, \hat{\imath}_t, \hat{d}_t, \hat{y}_t, \hat{w}_t, \lambda_t, E_t\hat{n}_{t+1}, E_t\hat{c}_{t+1}^m, E_t\hat{d}_{t+1}, E_t\lambda_{t+1}\right]'.$$

Finally, τ_t denotes the vector of rational expectations forecast errors:

$$\tau_t = \left[(\hat{n}_t - E_{t-1}\hat{n}_t), (\hat{c}_t^m - E_{t-1}\hat{c}_t^m), \left(\hat{d}_t - E_{t-1}\hat{d}_t\right), (\lambda_t - E_{t-1}\lambda_t)\right]',$$

and ε_t represents innovations to exogenous productivity disturbances.

The model is solved using the solution algorithm developed by Sims (2000) as adapted to sunspot equilibria by Lubik and Schorfheide (2003). In the case of indeterminacy, in addition to the fundamental technology shock, the manager observes an exogenous sunspot shock, ν_t, which influences dynamics of the key macroeconomic variables. Consistency with rational expectations requires that the sunspot is i.i.d. with $E_{t-1}\nu_t = 0$.

Because of the linear structure of the model, the forecast errors for the next period labor, level of technology, dividend, and manager's consumption can be expressed as function of two sources of uncertainty: the technology shock and the sunspot

$$\tau_t = \Phi_1\varepsilon_t + \Phi_2\nu_t$$

where Φ_1 and Φ_2 have dimension 4×1. The solution algorithm of Sims (2000) explicitly constructs a mapping from shocks to the expectation errors in (D.1). As shown above, when δ is sufficiently small, our model has at least one stable solution. If Φ_1 is uniquely determined by the parameters ϑ and $\Phi_2 = 0$, the model has a unique solution. This is the case of determinacy in which the propagation mechanism of the technology shocks is uniquely determined. Thus, sunspots do not affect equilibrium allocations; neither do they induce fluctuations. If Φ_1 is not uniquely determined by the parameters ϑ and Φ_2 is different from zero, the equilibrium is indeterminate. In this case, sunspot shocks can be interpreted as shocks to endogenous forecast errors. Detailed technical conditions for indeterminacy are developed in Lubik and Schorfheide (2003). In our model, it is the subset of ϑ related to the manager's compensation and risk aversion that is critical for indeterminacy; this is given by $\vartheta_\nu = [\eta_m, \theta, \delta, A]$.

E Appendix E: Optimal Contract when $\mu = 0$

While Theorem 2 applies for any value of μ (including $\mu = 0$), the following Theorem states that in the limiting case where $\mu = 0$, one can find another contract that is also optimal, but that is convex in the firm's dividend.[38]

Theorem 4 *Suppose that the measure of managers $\mu = 0$, and that η_s and η_m satisfy either $0 \leq \eta_s < 1$, $0 \leq \eta_m < 1$, or $1 < \eta_s$, $1 < \eta_m$. Then the contract of the form (12) with $A = 0$, $0 < \varphi = \Lambda_4^{1/\eta_m}$, $\delta = 1$, $\gamma = 1$, and $\theta = \frac{1-\eta_s}{1-\eta_m} > 0$ is optimal. If, in addition, $|\eta_m - 1| < |\eta_s - 1|$, then $\theta > 1$ and this contract is convex in $w_t n_t + d_t(f)$.*

Proof. The equilibrium conditions resulting from the model with a compensation contract (12) are given in (19)–(28), (4)–(5), and (11) holding with equality. To prove Theorem 4 we simply need to show that in the case that the contract's parameters take the values stated in the Theorem 4, the resulting equilibrium conditions imply the same equilibrium as the optimal plan characterized by equations (B.11)–(B.23). Note that in the case that the measure of managers $\mu = 0$ considered here, the optimal risk-sharing condition (B.11) need not be satisfied in an optimal equilibrium. When the contract satisfies the restrictions just mentioned, we obtain

$$\begin{aligned} c_t^m &= \varphi\left(w_t n_t + d_t\right)^{\frac{1-\eta_s}{1-\eta_m}} \\ c_t^s &= w_t n_t + d_t, \end{aligned}$$

so that (B.18), (B.19) hold and

$$c_t^m = \varphi\left(c_t^s\right)^{\frac{1-\eta_s}{1-\eta_m}}. \tag{E.1}$$

The variable x_t, given in (19), reduces to

$$x_t = \frac{1-\eta_s}{1-\eta_m}\Lambda_4^{\frac{1-\eta_m}{\eta_m(1-\eta_s)}}\left(c_t^m\right)^{\frac{\eta_m-\eta_s}{1-\eta_s}}$$

so that equation (18) reduces to

$$\left(c_t^m\right)^{\frac{\eta_m-\eta_s}{1-\eta_s}-\eta_m} = \beta E_t\left[\left(c_{t+1}^m\right)^{\frac{\eta_m-\eta_s}{1-\eta_s}-\eta_m} r_{t+1}\right],$$

when $\mu = 0$. Using (E.1) to replace c_t^m, this expression reduces to the consumer-worker-shareholder's optimal condition for intertemporal consumption allocation (B.15)

$$\left(c_t^s\right)^{-\eta_s} = \beta E_t\left[\left(c_{t+1}^s\right)^{-\eta_s} r_{t+1}\right].$$

All other conditions are identical to the remaining expressions characterizing the optimal plan, (3), (4), (5), and (B.17). The proposed contract is thus optimal when $\mu = 0$. ■

[38] A less general version of this optimal contract is considered in Danthine and Donaldson (2008a).

F Appendix F: Generalized Model with Manager Participating on Financial Markets

In the model of Section 2, the managers have no access to financial markets. They may smooth their consumption over time by making investment decisions in the firm that result in the desired income pattern, but have no other opportunity to borrow or lend. We now extend this model to allow the managers to buy or sell riskless bonds and demonstrate that the results obtained in Section 2 are not sensitive to our assumption that managers are excluded from financial markets.

The model presented below is essentially identical to the one of Section 2 augmented with (i) the possibility for managers to access the bond market, and (ii) one technical change incorporated so that log-linear approximation techniques can, in this more general setting, still provide reliable conclusions pertaining to local determinacy or explosiveness of the general equilibrium. This technical change incorporates tiny time variation in the agents' discount factors in order to retain stationarity of the bond holdings. In the absence of such time variation, when both managers and consumer-shareholder-workers have access the bond market, transitory productivity shocks will generally have a permanent effect on the agents' consumption to the extent that one group of agents may borrow from the other group on impact and make interest payments forever.

The flow of ideas in this section is straightforward. First we characterize the competitive equilibrium under bond trading in an environment where managers are compensated according to the contract specified in (12). We then describe the sets of parameter combinations which lead to equilibrium indeterminacy/instability. These sets are presented in a manner directly analogous to the regions described in the prior figures 1–4.

Accordingly, we begin by detailing the consumer-worker and manager problems for this expanded setting.

F.1 The Consumer-Worker-Shareholder's Problem

The representative consumer-worker-shareholder chooses processes for consumption c_t^s, labor supply n_t^s, bond holdings b_t^s, and equity holdings $z_t^e(f)$ to maximize his expected utility

$$E_0\left[\sum_{t=0}^{\infty}\chi_{t-1}\left(\frac{(c_t^s)^{1-\eta_s}}{1-\eta_s}-B\frac{(n_t^s)^{1+\zeta}}{1+\zeta}\right)\right] \tag{F.1}$$

subject to his budget constraint

$$c_t^s+\int q_t^e(f)z_{t+1}^e(f)df+q_t^b b_{t+1}^s=\int\left(q_t^e(f)+d_t(f)\right)z_t^e(f)df+w_t n_t^s+b_t^s. \tag{F.2}$$

As in Section 2, w_t denotes the competitive wage rate, q_t^b and q_t^e, are respectively bond and equity prices, d_t denotes the equity security's dividend. (We assume that each household is holding a fully diversified portfolio of shares of each firm in equal proportions so that the dividends d_t are aggregated over all firms.) Consistent with Ferrero, Gertler and Svensson (2010), the discount factor χ_t evolves according to $\chi_t=\beta_t\chi_{t-1}$, with $\chi_{-1}=1$, and

$$\beta_t\equiv\beta\left[1+\phi\left(\log\bar{c}-\bar{\vartheta}\right)\right]/\left[1+\phi\left(\log c_t-\bar{\vartheta}\right)\right], \tag{F.3}$$

where $c_t=c_t^s+\mu c_t^m$ corresponds to the aggregate consumption level but is treated by households and managers as exogenous, $\beta\in(0,1)$ is the steady-state discount factor, and ϕ is calibrated to a very small positive number and $\bar{\vartheta}$ is a constant. In the limit as $\phi \longmapsto 0$, $\chi_{t-1} \longmapsto \beta^t$.[39]

F.2 The Manager's Problem

The manager of firm f chooses processes for his own consumption, $c_t^m(f)$, hiring decisions, $n_t(f)$, investment $i_t(f)$ in the physical capital stock, $k_t(f)$, dividends, $d_t(f)$, and riskless bond holdings $b_t^m(f)$ to maximize her discounted expected utility

$$E_0 \left[\sum_{t=0}^{\infty} \chi_{t-1} \frac{c_t^m(f)^{1-\eta_m}}{1-\eta_m} \right] \tag{F.4}$$

subject to

$$\begin{aligned} c_t^m(f) + q_t^b b_{t+1}^m(f) &\leq g_t^m(f) + b_t^m(f) && \text{(F.5)} \\ d_t(f) &= y_t(f) - w_t n_t(f) - i_t(f) - \mu g_t^m(f) && \text{(F.6)} \\ y_t(f) &= (k_t(f))^{\alpha} (n_t(f))^{1-\alpha} e^{\lambda_t} && \text{(F.7)} \\ k_{t+1}(f) &= (1-\Omega) k_t(f) + i_t(f), \qquad k_0(f) \text{ given.} && \text{(F.8)} \end{aligned}$$

The manager's budget constraint (F.5) states that her consumption and the value of her newly purchased bonds can be no larger than her compensation, $g_t^m(f)$, plus the value of the bond holdings in the prior period. Equation (F.6) states that dividends, $d_t(f)$, are given by the free cash flows of the firm. Finally (F.7) specifies the firm's production technology, while (F.8) describes the firm's capital accumulation. The manager chooses to operate the firm provided that

$$E_0 \left[\sum_{t=0}^{\infty} \chi_{t-1} \frac{(c_t^m)^{1-\eta_m}}{1-\eta_m} \right] \geq \frac{\overline{u}^m}{1-\beta}. \tag{F.9}$$

F.3 The Decentralized Equilibrium

We next define the decentralized equilibrium in which the consumer-shareholder-worker chooses his own consumption, labor supply, bonds and equity holdings and in which the manager of firm $f \in [0,1]$ chooses her own stream of consumption, makes decisions about physical investment, hiring, and dividend payouts, and is paid according to the following pre-specified compensation contract:

$$g^m(w_t n_t, d_t, d_t(f)) = A + \varphi \left[\delta (w_t n_t + d_t)^{\gamma} + d_t(f) - d_t \right]^{\theta} \tag{F.10}$$

for non-negative coefficients $A, \delta, \varphi, \gamma$ and θ. The relevant contract is identical to (12).

The consumer-shareholder-worker chooses $c_t^s, n_t^s, z_{t+1}^e(f), b_{t+1}^s$ to maximize his utility function (F.1) subject to his budget constraint (F.2) and (F.3). This yields the first-order necessary condi-

[39] This formulation of the discount factor incorporates the stimulative effect on individual consumption of an increase in average consumption, as in Uzawa (1968). However the parameter ϕ is calibrated to such a small value that this effect is negligible. As mentioned above, it merely serves as a technical device to guarantee a unique steady state in the case of incomplete financial markets across groups of agents. One can alternatively obtain a such a unique steady state by assuming a constant discount factor β, but introducing a debt-elastic interest rate premium in the budget constraints (F.2) and (F.5) below, as in Benigno (2001), Kollmann (2002), Schmitt-Grohe and Uribe (2003), and Justiniano and Preston (2010).

tions

$$
\begin{aligned}
\left(c_t^s\right)^{-\eta_s} w_t &= B\left(n_t^s\right)^{\zeta} && \text{(F.11)}\\
q_t^e(f) &= E_t\left[\beta_t \frac{\left(c_{t+1}^s\right)^{-\eta_s}}{\left(c_t^s\right)^{-\eta_s}}\left(q_{t+1}^e(f)+d_{t+1}(f)\right)\right] && \text{(F.12)}\\
q_t^b &= E_t\left[\beta_t \frac{\left(c_{t+1}^s\right)^{-\eta_s}}{\left(c_t^s\right)^{-\eta_s}}\right] && \text{(F.13)}
\end{aligned}
$$

which are identical to (3)–(5), except for β_t being allowed to vary.

Each manager in turn decides whether to manage the firm or instead whether receive her reservation utility. The measure μ of managers who work in firm f chooses $c_t^m(f), n_t(f), k_{t+1}(f), b_{t+1}^m(f)$, $d_t(f)$ to maximize their utility (F.4) subject to the restrictions (F.5)–(F.8) and the compensation contract (F.10). We will assume that the compensation is sufficiently generous for each manager's participation constraint (F.9) to be satisfied, so that they all decide to work in a firm. The Lagrangian for the representative manager's problem in firm f can be expressed as

$$
\begin{aligned}
\mathcal{L} = \; & E_0\left\{\sum_{t=0}^{\infty}\chi_{t-1}\left(\frac{\left(c_t^m(f)\right)^{1-\eta_m}}{1-\eta_m}\right.\right.\\
& +\Lambda_{1t}(f)\Big[\left(k_t(f)\right)^{\alpha}\left(n_t(f)\right)^{1-\alpha}e^{\lambda_t}-w_t n_t(f)-k_{t+1}(f)+(1-\Omega)k_t(f)\\
& -\mu\left(A+\varphi\left[\left(\delta w_t n_t+d_t\right)^{\gamma}+d_t(f)-d_t\right]^{\theta}\right)-d_t(f)\Big]\\
& \left.\left.+\Lambda_{2t}(f)\left[A+\varphi\left[\left(\delta w_t n_t+d_t\right)^{\gamma}+d_t(f)-d_t\right]^{\theta}+b_t^m(f)-c_t^m(f)-q_t^b b_{t+1}^m(f)\right]\right)\right\}.
\end{aligned}
$$

The necessary first-order conditions with respect to $c_t^m(f), n_t(f), b_{t+1}^m(f), k_{t+1}(f), d_t(f)$ are

$$
\begin{aligned}
\Lambda_{2t}(f) &= \left(c_t^m(f)\right)^{-\eta_m} && \text{(F.14)}\\
w_t &= (1-\alpha)\,y_t(f)/n_t(f) && \text{(F.15)}\\
q_t^b &= E_t\left[\beta_t\frac{\Lambda_{2t+1}(f)}{\Lambda_{2t}(f)}\right] && \text{(F.16)}\\
\Lambda_{1t}(f) &= E_t\left\{\beta_t\Lambda_{1t+1}(f)\left[1-\Omega+\alpha y_{t+1}(f)/k_{t+1}(f)\right]\right\} && \text{(F.17)}
\end{aligned}
$$

and

$$
\Lambda_{1t}(f)=\Lambda_{2t}(f)\frac{x_t(f)}{1+\mu x_t(f)} \tag{F.18}
$$

for all f, where $x_t(f)$ is again defined as in (17), so that

$$
x_t(f)=\theta\varphi\left[\left(\delta w_t n_t+d_t\right)^{\gamma}+d_t(f)-d_t\right]^{\theta-1}. \tag{F.19}
$$

F.3.1 Equilibrium definition

The decentralized equilibrium is a set of processes $\{c_t^s, c_t^m, n_t^s, n_t^f, b_t^s, b_t^m, z_t^e, i_t, y_t, k_t, r_t, w_t, q_t^b, q_t^e, d_t,$ $\beta_t, \chi_{t-1}\}$ such that:

1. The first-order conditions (F.11)–(F.19) are satisfied together with the constraints (F.2),

(F.3), (F.5)–(F.8), (F.9), $\chi_t = \beta_t \chi_{t-1}$, all holding with equality, and the transversality condition: $\lim_{t\to\infty} \chi_{t-1} u_1^m (c_t^m) k_{t+1} = 0$, for any given initial k_0, and for $\chi_{-1} = 1$.

2. The labor, goods and capital markets clear: $n_t^s = n_t$; $y_t = i_t + c_t^s + \mu c_t^m$; and investors hold all outstanding equity shares, $z_t^e = 1$, and all other assets (one period bonds) are in zero net supply, $b_t^s + \mu b_t^m = 0$.

By imposing market clearing relationships on the necessary and sufficient first order conditions detailed above, we can also fully characterize the equilibrium as the set of processes $\{c_t^s, c_t^m, n_t^s, n_t^f, b_t^s, b_t^m, z_t^e, i_t, y_t, k_t, r_t, w_t, q_t^b, q_t^e, d_t, \beta_t\}$ which satisfy:

$$(c_t^s)^{-\eta_s} w_t = B (n_t)^\zeta \tag{F.20}$$

$$q_t^e = E_t \left[\beta_t \frac{\left(c_{t+1}^s\right)^{-\eta_s}}{\left(c_t^s\right)^{-\eta_s}} \left(q_{t+1}^e + d_{t+1}\right) \right] \tag{F.21}$$

$$q_t^b = E_t \left[\beta_t \frac{\left(c_{t+1}^s\right)^{-\eta_s}}{\left(c_t^s\right)^{-\eta_s}} \right] \tag{F.22}$$

$$q_t^b = E_t \left[\beta_t \frac{\left(c_{t+1}^m\right)^{-\eta_m}}{\left(c_t^m\right)^{-\eta_m}} \right] \tag{F.23}$$

$$r_t \equiv 1 - \Omega + \alpha y_t / k_t \tag{F.24}$$

$$w_t = (1-\alpha) y_t / n_t \tag{F.25}$$

$$c_t^s = w_t n_t + d_t - \mu \left(b_t^m - q_t^b b_{t+1}^m \right) \tag{F.26}$$

$$c_t^m = A + \varphi (\delta w_t n_t + d_t)^{\gamma\theta} + b_t^m - q_t^b b_{t+1}^m \tag{F.27}$$

$$d_t = y_t - w_t n_t - i_t - \mu \left(A + \varphi (\delta w_t n_t + d_t)^{\gamma\theta} \right) \tag{F.28}$$

$$y_t = k_t^\alpha n_t^{1-\alpha} e^{\lambda_t} \tag{F.29}$$

$$k_{t+1} = (1-\Omega) k_t + i_t \tag{F.30}$$

$$\beta_t \equiv \beta \left[1 + \phi \left(\log \bar{c} - \bar{\vartheta}\right)\right] / \left[1 + \phi \left(\log c_t - \bar{\vartheta}\right)\right] \tag{F.31}$$

$$(c_t^m)^{-\eta_m} = E_t \left[\beta_t \left(c_{t+1}^m\right)^{-\eta_m} \left(\frac{1 + \mu x_t}{1 + \mu x_{t+1}} \right) \left(\frac{x_{t+1}}{x_t} \right) r_{t+1} \right] \tag{F.32}$$

$$x_t = \theta\varphi (\delta w_t n_t + d_t)^{\gamma(\theta-1)} \tag{F.33}$$

where k_0 is given, and where again we assume that the manager's compensation is large enough for his participation constraint (F.9) to be satisfied.

F.3.2 Approximating the Decentralized Equilibrium around the Deterministic Steady State

We assume that in the steady state, consumer-shareholder-workers and managers hold no nominal bonds ($\bar{b}^m = 0$). As a result, the steady state in this economy is the same as the model of Section 3. Denoting the steady state value of a variable with an overhead bar, it is defined as the solution to the following set of equations: $\bar{c}^m = A + \left(\varphi\delta\bar{w}\bar{n} + \varphi\bar{d}\right)^{\gamma\theta}$; $\bar{w}\bar{n} = (1-\alpha)\bar{y}$; $\bar{y} = \bar{k}^\alpha \bar{n}^{1-\alpha}$; $\Omega\bar{k} = \bar{\imath}$; $\bar{x} = \varphi\theta\left(\delta\bar{w}\bar{n} + \bar{d}\right)^{\gamma(\theta-1)}$; $\bar{r} = \alpha\bar{y}/\bar{k} + 1 - \Omega$; $\beta^{-1} = \bar{r}$; $\bar{c}^s = \bar{w}\bar{n} + \bar{d}$ and $(\bar{c}^s)^{-\eta_s} \bar{w} = B\bar{n}^\zeta$. Denoting the log-deviations from that steady state value with a ˆ, and defining $\hat{b}_t^m \equiv b_t^m / \bar{c}^m$, we can approximate the model's dynamics by the following log-linearized equilibrium conditions:

$$\begin{aligned}
\hat{c}_t^m &= \Xi\left[\delta\omega\hat{y}_t + (1-\omega)\,\hat{d}_t\right] + \hat{b}_t^m - \beta\hat{b}_{t+1}^m \\
\hat{c}_t^s &= \omega\hat{y}_t + (1-\omega)\,\hat{d}_t - \mu\frac{\bar{c}^m}{\bar{c}^s}\left(\hat{b}_t^m - \beta\hat{b}_{t+1}^m\right) \\
\Omega\frac{\bar{k}}{\bar{y}}\hat{\imath}_t &= \left(\alpha - \mu\frac{\bar{c}^m}{\bar{y}}\Xi\delta\omega\right)\hat{y}_t - \left(\frac{\bar{d}}{\bar{y}} + \mu\frac{\bar{c}^m}{\bar{y}}\Xi\,(1-\omega)\right)\hat{d}_t \\
\hat{y}_t &= \lambda_t + \alpha\hat{k}_t + (1-\alpha)\,\hat{n}_t \\
\hat{w}_t &= \hat{y}_t - \hat{n}_t \\
\hat{r}_t &= (1-\beta\,(1-\Omega))\left(\hat{y}_t - \hat{k}_t\right) \\
\hat{w}_t &= \eta_s\hat{c}_t^s + \zeta\hat{n}_t \\
\hat{\beta}_t &= -\tilde{\phi}\left(\hat{c}_t^s + \mu\frac{\bar{c}^m}{\bar{c}^s}\hat{c}_t^m\right) \\
\hat{q}_t^b &= \hat{\beta}_t - \eta_s E_t\hat{c}_{t+1}^s + \eta_s\hat{c}_t^s \\
\hat{q}_t^b &= \hat{\beta}_t - \eta_m E_t\hat{c}_{t+1}^m + \eta_s\hat{c}_t^m \\
\hat{x}_t &= \left(\frac{\theta-1}{\theta}\right)\frac{\bar{c}^m}{\bar{c}^m - A}\left(\hat{c}_t^m - \hat{b}_t^m + \beta\hat{b}_{t+1}^m\right) \\
\hat{k}_{t+1} &= (1-\Omega)\,\hat{k}_t + \Omega\hat{\imath}_t
\end{aligned}$$

and the log-linearized Euler equation for the manager's consumption

$$-\eta_m\hat{c}_t^m = E_t\left[\hat{\beta}_t - \eta_m\hat{c}_{t+1}^m + \frac{1}{1+\mu\bar{x}}\left(\hat{x}_{t+1} - \hat{x}_t\right) + \hat{r}_{t+1}\right],$$

where

$$\omega \equiv \frac{\bar{w}\bar{n}}{\bar{w}\bar{n} + \bar{d}}, \qquad \Xi \equiv \frac{\gamma\theta\,(1 - A/\bar{c}^m)}{\delta\omega + 1 - \omega} > 0, \qquad \tilde{\phi} \equiv \frac{\phi\bar{c}^s/\bar{c}}{1 + \phi\left(\log\bar{c} - \bar{\vartheta}\right)} > 0.$$

As in Section 3, the set of equations above will form the basis for our characterization of the regions of indeterminacy.

F.3.3 Regions of indeterminacy

Figures 5, 6, and 7 present the regions of indeterminacy/instability for the same specific scenarios, respectively, that underlie Figures 1, 2, and 3 with the addition of bond trading by the managers. In all cases the contract form remains as in (12). Subject to this contract form, the regions identify sets of parameter combinations that give rise to indeterminacy/instability, as before.

F.4 Is Bond Trading Important?

Under the optimal contract, the resulting equilibrium is a Pareto optimum and thus represents an allocation that cannot be improved upon for both managers and shareholder-workers. The presence of voluntary bond trading between agents can neither improve upon nor detract from the Pareto allocation. In other words, the optimal contract (12) in conjunction with parameter choices specified in Theorem 1, remains optimal even if the structure of the economy is generalized to admit

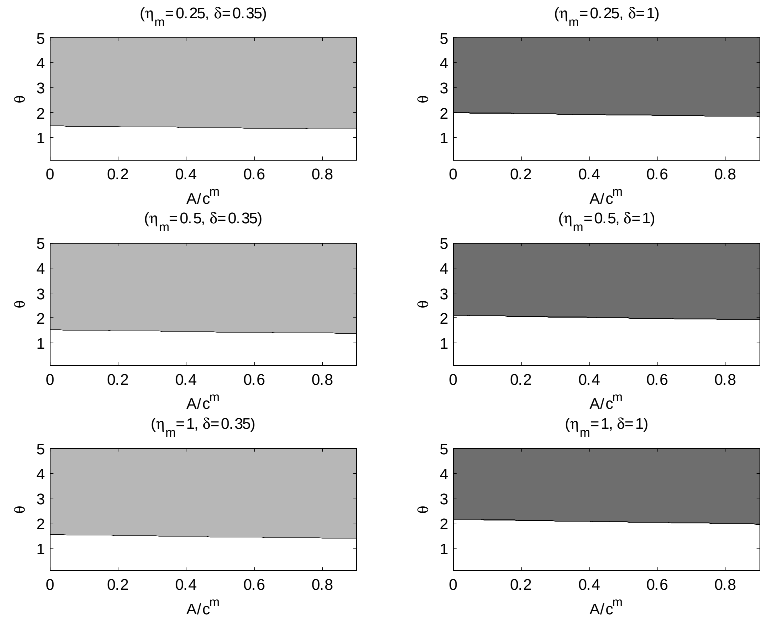

Figure 5: Analogue of Figure 1 but with the admission of bond trading.

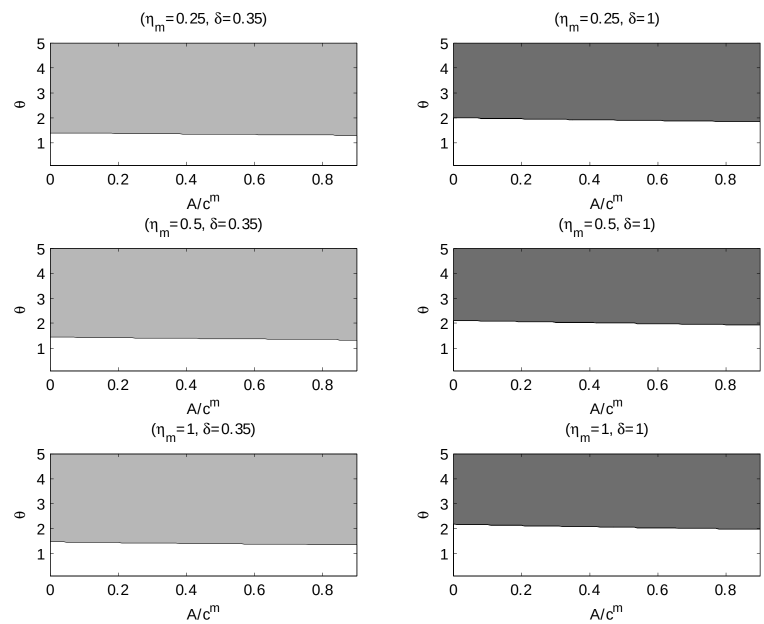

Figure 6: Analogue of Figure 2 but with the admission of bond trading.

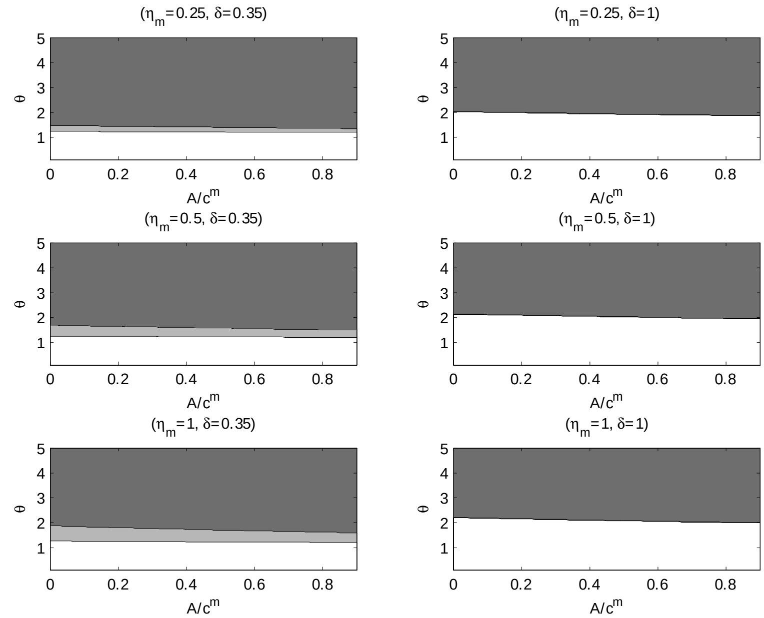

Figure 7: Analogue of Figure 3 but with the admission of bond trading.

bond trading. The task of this Appendix is to verify this claim formally. We do so by first deriving the solution to the planner's problem (the complete markets solution), and then demonstrating that the equilibrium allocation under the optimal contract and zero bond trading coincides with it. In this sense optimal contracting can be viewed as a substitute for complete markets and thus more than a substitute for bond trading.

The planner's problem is as follows:

$$\max_{\{c_t^m, c_t^s, n_t^s, k_{t+1}\}} E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\phi\mu \frac{(c_t^m)^{1-\eta_m}}{1-\eta_m} + \frac{(c_t^s)^{1-\eta_s}}{1-\eta_s} - B\frac{n_t^{1-\zeta}}{1-\zeta} \right] \right)$$

subject to

$$c_t^s + \mu c_t^m + k_{t+1} - (1-\Omega)\, k_t = y_t = k_t^\alpha n_t^{1-\alpha} \lambda_t.$$

It is well known that the necessary and sufficient first order conditions for the above problem are:

$$\phi\,(c_t^m)^{-\eta_m} = (c_t^s)^{-\eta_s} \tag{F.34}$$

$$B n_t^{-\zeta} = (c_t^s)^{-\eta_m}\, k_t^\alpha n_t^{-\alpha} \lambda_t \tag{F.35}$$

$$(c_t^m)^{-\eta_m} = \beta E_t \left[\left(c_{t+1}^m\right)^{-\eta_m} \left(\alpha k_{t+1}^\alpha n_{t+1}^{1-\alpha} \lambda_{t+1} + 1 - \Omega \right) \right]. \tag{F.36}$$

Together with the constraints

$$\mu c_t^m + c_t^s + i_t = y_t, \quad \text{and} \tag{F.37}$$

$$k_{t+1} = (1-\Omega)\, k_t + i_t, \tag{F.38}$$

these five equations completely and uniquely determine the real Pareto allocation. Given the optimal contract form and parameter values and no-bond-trading, we will next demonstrate that the equilibrium characterization (F.20)–(F.33) guarantees these exact same relationships. Its real allocation must thus be identical.

By (F.25) and (F.20), (F.35) is satisfied. By (F.25), (F.32) and (F.33), and recognizing that for the optimal contact, $\theta = 1$ so that $x_t = \varphi$, (F.32) is identical to (F.36). By construction (F.38) and (F.30) are identical. By (F.26), (F.27) and the definition of dividends, (F.28), (F.37) is guaranteed. Lastly, for the optimal parameter choices ($A = 0$, $\theta = 1$, $\gamma = \frac{\eta_s}{\eta_m}$) and no bond trading ($b_t^m = b_t^s = 0$), (F.26) and (F.27) guarantee optimal risk sharing, (F.34).

In this setting, therefore, optimal contracting substitutes for market completeness, and thus for bond trading as well. For contracts parameterized non-optimally, however, bond trading may expand the parameter set leading to indeterminacy/instability.

References

[1] Benigno, Pierpaolo, 2001, "Price Stability with Imperfect Financial Integration," unpublished, New York University.

[2] Ferrero, Andrea, Mark Gertler, and Lars E.O. Svensson (2010), "Current Account Dynamics and Monetary Policy," in Jordi Galí and Mark Gertler (eds.), *International Dimensions of Monetary Policy*, Chicago: University of Chicago Press.

[3] Justiniano, Alejandro and Bruce Preston, 2010, "Monetary Policy and Uncertainty in an Empirical Small Open Economy Model," *Journal of Applied Econometrics*, 25: 93-128.

[4] Kollman, Robert, 2002, "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles," *Journal of Monetary Economics*, 49, 989-1015.

[5] Schmitt-Grohe, Stephanie and Martin Uribe, 2003, "Closing Small Open Economy Models," *Journal of International Economics*, 61, 163-195.

[6] Uzawa, Hirofumi (1968), "Time Preference, the Consumption Function and Optimum Asset Holdings," in James Wolfe (ed.), *Value, Capital and Growth: Papers in Honor of Sir John Hicks*, Edinburgh: The University of Edinburgh Press.